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Numerical simulations for study of optimal fiscal policies in a contemporary dynamic dual economy

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Abstract

Numerical simulations are undertaken for the theory developed in Das and Murty (2022) to study optimal fiscal policies in a contemporary dual economy characterised by migration and skill acquisition by labour force in the informal agricultural sector who contribute to human capital in the formal manufacturing sector. Fiscal policy is successful in mitigating inequalities due to skewed distribution of initial endowments and the unbalanced nature of growth exhibited by our dual economy. While a standard interior solution, where there is an equalisation of net social marginal benefits and costs, is obtained for the optimal tax on human capital, social welfare is decreasing in taxation of consumption and physical capital. The optimal tax on physical capital is defined by the lower bound on the set of tax rates on physical capital that ensure that its relative shadow price and the redistributive transfer are well-defined and could be positive. Reductions in the productivity differential between skilled and unskilled labour entail decreases in the optimal tax on human capital, increases in its long-run growth, and increases in migration and skill formation by the migratory labour force. Lastly, our simulations indicate that social welfare maximising fiscal policies prefer to employ capital taxation as sparingly as possible for meeting the redistributive goal in a bid to promote also the growth objective of the government.

Numerical simulations for study of optimal fiscal policies in a contemporary dynamic dual economy

1 Introduction.

In an earlier working paper (Das and Murty (2022), henceforth referred to as DM), we proposed a model of a dynamic dual economy in the context of a labour-abundant contemporary developing economy with a formal (interpreted as an aggregated urban manufacturing and services) sector and an informal (to be interpreted as the rural agricultural) sector. We believe that a persistent dual structure arises in such economies primarily due to (i) abundance of the labour resource in such economies naturally induces differences in the technologies and the nature of inputs employed in the two sectors, (ii) the differences in the demand patterns for the goods produced by the formal and informal sectors, (iii) the essentiality of consumption of the agricultural good in amounts in excess of the subsistence level of consumption, and (iv) limited factor mobility between the two sectors. In particular, the unskilled labour in the informal sector can migrate and contribute to the skilled labour force/human capital in the formal sector by investing in costly skill acquisition. The more is the skill acquisition by the migrating labour force, the more is their contribution to the human capital in the formal sector. The type of physical capital employed in the formal sector is assumed to be qualitatively different from the type employed in the informal sector. However, both types of physical capital compete for investment expenditure in each household's budget.

We argue that the dual structure of such developing economies will continue to persist as long as these countries are characterised by an abundance of unskilled labour resource. This demographic dividend (or the increase in the share of population in the working age group relative to the dependent population) is a feature common to many contemporary developing economies such as India. Moreover, this persistence of duality is consistent with endogenous but unbalanced growth of the two sectors with the government contributing to the production in both sectors by incurring sector-specific public infrastructural expenditures. The model is developed to understand the role of fiscal policy in mitigating the inequalities in social welfare that may arise in such dual economies.

In this paper, the theory developed in DM is subjected to some numerical simulations to study the features of social-welfare maximising fiscal policies that can be derived from this model. As is empirically observed, only incomes earned in the formal sector are taxed, while both formal and informal households are subjected to the consumption tax. In addition, the economic rent generated by provision of public infrastructure by the government in the formal sector is also assumed to be taxed.[1] The total revenue so generated is employed to finance a transfer to the informal sector and expenditures on public infrastructure and a public good that is consumed by both the types of households.

Given a vector of the relevant parameters of the model and the initial distributions of endowments, a macroeconomics tax equilibrium of this model is computed for each configuration of capital and consumption tax rates by adapting and extending the methodology developed in Turnovsky (1996). Expenditures on public infrastructure in these two sectors are endogenously determined at a tax equilibrium as are the proportion of migratory labour force in the total labour force employed in the informal sector, and the average level of skill acquired by the migratory labour force. In this model, these variables along with the relative shadow prices of all types of capital modelled lack transitional dynamics and are constant over time.[2]

The utilitarian form of the general iso-elastic inter-temporal social welfare function that was derived in DM, is employed to compute the social welfare at the macroeconomic tax equilibrium corresponding to every configuration of capital and consumption tax rates for which the relative shadow prices of capital are well defined.[3] Maximisation of social welfare involves choosing the vector of tax rates and hence the macroeconomic tax equilibrium that leads to the greatest level of inter-temporal social welfare.

Given the base levels of parameters and distributions of initial endowments chosen by us for our numerical simulations, we find that the inter-temporal social welfare function is strictly concave and achieves a maximum with respect to the tax rate on human capital. Thus, welfare maximisation yields an interior solution for the tax rate on human capital, with the net social marginal benefit with respect to taxation of human capital being negatively sloped and taking a value of zero at the second-best optimum (solution of the social welfare maximisation problem). In particular, at this optimum, there is an inter-temporal marginal benefit to the informal sector household from taxing human capital that is exactly equal to the inter-temporal marginal cost from taxing human capital to the formal sector household. Hence, the net social marginal benefit from taxation of human capital at the social optimum is zero.

[1] Participants in the informal sector are assumed to be self employed. Since incomes generated in this sector are untaxed, the economic rent from public infrastructure in this sector accrues as additional income to participants in the informal sector.

[2] This follows from the use of the methodology developed in Turnovsky (1996).

[3] Our numerical analysis finds that there are tax rates for which the the relative shadow prices of capital may not be well defined, so tax equilibria corresponding to these rates of taxes do not exist

An alternative decomposition of the net social marginal benefit of taxing human capital at optimum is also obtained whereby the direct and indirect impacts of taxing human capital on social welfare are computed. The indirect impacts include the impacts of taxing human capital on social welfare due to the fact that such taxation can impinge on the equilibrium levels of migration, skill formation by the migratory labour force, and the net return to human capital and its rate of accumulation. The direct effects of taxing human capital on social welfare include the effect it has on tax revenue and, hence, on the levels of the redistributive transfer to the informal sector, the public infrastructure, and the public good that are financed by this revenue. While the impact on migration and skill formation by migratory labour force affects the welfare of only the household in the informal sector, welfare of both types of households will be affected by the impacts of human capital taxation on the net return to human capital and the rate of investment in human capital, both of which will, in-turn, impinge on its long-run equilibrium rate of growth.

In a comparative-static study we find that, as the productivity of unskilled labour in the informal sector increases (intuitively, as the productivity differential between skilled and unskilled workers decreases), the optimal tax on human capital falls, and the long-run rate of growth of human capital increases as also do the share of migration in the labour force employed in the informal sector and the skill factor acquired by the migrating labour force. Although, we find that many countervailing forces operate, in our numerical example, there is a net increase in the share of migration in the labour force employed in the informal sector primarily due to the increase in the relative shadow price of human capital, which is induced by a fall in the optimal tax on human capital when there is an increase in the productivity of unskilled labour. The increase in the relative shadow price of human capital incentivises the unskilled labour force to migrate and contribute more to the human capital in the formal sector, while the income effect due to increased productivity of the unskilled labour makes acquisition of greater level of skills more affordable for the migrating labour force.

The initial distributions of endowments are chosen in our numerical simulations so that the informal sector household is initially impoverished relative to the formal sector household. We find that the optimal fiscal policy implies significant redistribution from the formal to the informal sector household through the use of the transfer instrument. However, the mode of financing the transfer changes as productivity of the unskilled labour increases. Increases in the productivity of the unskilled labour by allowing the optimal tax of human capital to fall promote higher rates of growth of human capital, with the redistributive transfer being increasingly financed by taxation of the economic surplus/profit generated by public infrastructure in the formal sector. This shows that in institutions where alternative instruments for generating tax revenue exist, optimal capital taxation policies aim more and more to promote the growth objectives, with the redistributive objectives of the government being met more and more by instruments that do not directly tax capital. Thus, capital taxation tends to be used as sparingly as possible.

In contrast to the case of taxation of human capital reported above, inter-temporal social welfare is found to be decreasing in both the taxation of consumption and the taxation of physical capital in the formal sector. The net social marginal benefit from consumption taxation is negative – the marginal social cost of employing this instrument is the reductions in consumption by *both* types of households – formal and informal – that it induces. This outweighs the marginal social benefit from the increases in the tax revenue that it generates for financing the redistributive transfer and other public expenditures. Thus, the optimal tax on consumption is a corner solution of the social welfare maximisation problem, *i.e.,* it is zero.

The net social marginal benefit from taxation of physical capital employed by the formal sector is negative primarily because of the negative impact that this form of taxation has on the net return and the rate of investment in physical capital and hence on its long-term growth. But the optimal tax on physical capital employed in the formal sector need not be a conventional corner solution of the social welfare maximisation problem, *i.e.,* it need not be zero, even though social welfare is decreasing in this tax rate. The optimal tax is determined by the lower bound on the set of tax rates that ensure that the relative shadow price of the formal-sector physical capital is well-defined and the redistributive transfer is non-negative.[4] As the productivity of physical capital in the formal sector increases (intuitively, as the productivity differential between the formal and informal sector physical capital increases), we find that this lower bound also increases. Hence, the optimal tax on physical capital employed in the formal sector also increases with increases in its productivity.

Lastly, we find that our simulated dual economy is characterised by unbalanced growth, with the long-run growth of the informal sector lagging behind the long-run growth of the formal sector. The former is determined by the bigger of the long-run rates of growth of the two forms of capital – human or physical – employed in the formal sector, while the latter is determined by the long-run rate of growth of consumption.[5]

[4]A tax equilibrium with non-negative redistributive transfer does not exist for tax rates on physical capital that are below this lower bound. We find many instances in our numerical simulations where this lower bound on tax rates is strictly positive.

[5]Which has been shown to be the same for both the formal and the informal sector goods.

In Section 2 we spell out the features of the special case of the general model presented in DM that we employ for our numerical simulations. In Section 3, we parametrise the tax equilibria of such an economy using alternative economic variables and present some comparative static results pertaining to the tax equilibria. In Section 4, the second-best social welfare maximisation problem is posed and the various possible forms that its solution takes in our numerical simulations are discussed. Comparative statics of social welfare maximisation are derived to study how optimal fiscal policy changes with changes in the productivity differentials across the formal and informal sectors. Section 5 presents the values of the base parameters and the distributions of initial endowments employed in our numerical simulations. It also describes the computation methodology used to perform our numerical simulations. Section 6 presents the results from our numerical simulations and provides some interpretations of these. We conclude in Section 7.

It is to noted that this work follows the same notation that is employed in DM. The reader is hence recommended to refer to DM for details on this.

2 A special case.

For a sharper understanding of the nature of optimal fiscal policies in the dual economy we constructed in DM, let's focus on the special case of a pure formal sector, *i.e,* the case where the formal sector household does not own endowments of the resources used in the informal sector, *i.e.,* where $L(0) = l(0) = K_{21}(0) = 0$. This implies that the formal sector household does not derive income from informal sector employment. To simplify the analysis further, we make the following additional assumptions: the tax rate on the returns from holding bond, τ_b, is not a policy instrument used by the government, *i.e.,* $\tau_b = 0$ and the physical capital depreciation rates in the formal and informal sectors are both zero, *i.e.,* $\delta_{K_1} = \delta_{K_2} = 0$.

In addition to the initial endowments in this economy, $B_m(0)$, $L_m(0)$, $l_m(0)$, $K_{1m}(0)$, $K_{2m}(0)$, and $H_m(0)$ for $m = 1, 2$, the following parameters of the problem discussed in DM will also be held fixed throughout the analysis:[6]

$$\mathcal{P} = \langle \rho, \epsilon, \varphi, \theta, \pi, A_1, A_2, \alpha_2, \alpha_H, \eta_2, \eta_1, \mu_1, \mu_2, r_b, n, g_c \rangle .$$

Comparative statics will be conducted with reference to the remaining two parameters in our model, namely, the productivity of unskilled labour, α_L, and the productivity of the physical

[6]Formal and informal sector households are indexed, respectively, by $m = 1$ and $m = 2$.

capital in the formal sector, α_1. Holding the productivity of skilled labour/human capital, α_H, fixed, changes in α_L will reflect productivity differentials between the skilled formal sector labour-force and the unskilled informal sector labour force. Similarly, holding the productivity of physical capital in the informal sector, α_2, fixed, changes in α_1 will reflect productivity differentials between capital in the formal and informal sectors. We are interested in studying the sensitivity of the socially optimal fiscal (especially redistributive) policy to changes in the productivity differentials between the formal and informal sectors.

Denote the vector of tax rates that can vary in this analysis as $\tau = \langle \tau_w, \tau_k, \tau_c \rangle \in [0,1]^3$ and the net wage and rental rates in the formal sector, respectively, as $\hat{w}_H^N$ and $\hat{r}_1^N$. These are given by:

$$\begin{aligned} \hat{w}_H^N &= (1-\tau_w)w_H = (1-\tau_w)\alpha_H =: \hat{w}_H^N(\tau, \alpha_L, \alpha_1, \mathcal{P}) \\ \hat{r}_1^N &= (1-\tau_k)r_1 = (1-\tau_k)\alpha_1 =: \hat{r}_1^N(\tau, \alpha_L, \alpha_1, \mathcal{P}), \end{aligned} \tag{1}$$

where the gross returns on human and physical capital in the formal sector at a macroeconomic tax equilibrium are given by $w_1 = \alpha_H$ and $r_1 = \alpha_1$, recalling that good 1, which is produced in the formal sector, is assumed to be the numeraire commodity. The equilibrium price of good 2 is a fixed constant given by $p = \frac{r_b}{\alpha_2}$. Hence, the equilibrium gross returns on unskilled labour and physical capital employed in the informal sector are given by $w_2 = p\alpha_L$ and and $r_2 = p\alpha_2$, respectively.

In DM investments as proportions of physical and human capital stocks employed in the formal sector at a macroeconomic tax equilibrium are given, respectively, by

$$\frac{I_{k_1 m}}{K_{1m}} = \frac{\lambda_{k_1 b} - 1}{\pi} \quad \text{and} \quad \frac{I_{hm}}{H_m} = \frac{\lambda_{hb} - 1}{\theta}$$

where $\lambda_{k_1 b}$ and λ_{hb} are, respectively, the shadow prices of physical and human capital employed in the formal sector, equilibrium values of which are given by[7]

$$\begin{aligned} \lambda_{hb} = \hat{\lambda}_{hb}(\hat{w}_H^N, \mathcal{P}) &= \left(1 + r_b\theta\right) - \sqrt{(1+r_b\theta)^2 - (1 + 2\theta\hat{w}_H^N)} =: \lambda_{hb}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \\ \lambda_{k_1 b} = \hat{\lambda}_{k_1 b}(\hat{r}_1^N, \mathcal{P}) &= 1 + \pi r_b - \sqrt{(1+\pi r_b)^2 - (1 + 2\pi\hat{r}_1^N)} =: \lambda_{k_1 b}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \end{aligned} \tag{2}$$

Thus, these shadow prices exhibit no transitional dynamics, *i.e.*, they are constant over time and across both types of households at a macroeconomic tax equilibrium. The shasow prices

[7]Here we note from (1) that w_H^N and r_1^N are functions of $\langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$.

of human and physical capital employed in the formal sector are increasing in the net returns to human and physical capital Further, DM define γ as the difference between the proportion of human capital that is invested and the rate of growth of the the working population in the informal sector:

$$\gamma = \gamma(\tau_w, \mathcal{P}) \;=\; \frac{\lambda_{hb}(\tau_w, \mathcal{P}) - 1}{\theta} - n.$$

It was shown that, provided $\gamma > 0$, the rates of growth of physical and human capital employed in the formal sector at a macroeconomic equilibrium are given by the proportions of human and physical capital invested in this sector. These are denoted, respectively, by ψ_{k_1} and ψ_H and are given by:

$$\begin{aligned} \psi_{k_1} &= \hat{\psi}_{k_1}(\lambda_{k_1 b}, \mathcal{P}) = \frac{\lambda_{k_1 b} - 1}{\pi} = \psi_{k_1}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \\ \psi_H = \gamma + n &= \hat{\psi}_H(\lambda_{hb}, \mathcal{P}) = \frac{\lambda_{hb} - 1}{\theta} = \psi_H(\tau, \alpha_L, \alpha_1, \mathcal{P}) \quad \text{if} \quad \gamma > 0 \end{aligned} \tag{3}$$

3 Characterising the tax equilibria and study of tax-equilibrium comparative statics.

In this section, we first parametrise the tax equilibria of the system using (i) some relevant economic variables that include the extent of migration ($\kappa = \frac{X}{L}$ defined as the amount of migration as a proportion of the net unskilled labour force employed in the informal sector), skill factor level (β), the long-run growth rates of physical and human capital in the formal sector (ψ_H, ψ_{k_1}), and the net rates of return on physical and human capital employed in the formal sector ($\hat{w}_H^N$, $\hat{r}_1^N$); (ii) fiscal policy variables such as various tax rates, $\tau = \langle \tau_w, \tau_k, \tau_c \rangle \in [0,1]^3$; (iii) the parameters of the system, $\langle \alpha_L, \alpha_1, \mathcal{P} \rangle$; and (iv) the initial endowments. A tax equilibrium can be associated with any configuration of these. Next we note that the tax equilibrium values of the economic variables in (i) are themselves functions of variables and parameters in (ii), (iii), and (iv) in our model. Hence, tax equilibria can ultimately be parametrised by fiscal policy variables and other parameters in (ii), (iii), and (iv). A tax equilibrium can be computed for any configuration of these provided conditions that ensure that shadow prices are well defined for the human and physical capital resources. Further, comparative static analyses are conducted that show how economic variables in (i) are impacted by fiscal policy variables in (ii) and two of the parameters in (iii) – namely, productivities of unskilled labour in the informal sector and physical capital in the formal sector.

To facilitate the analysis in this section, we denote $s := \langle \kappa,\ \beta,\ \psi_H,\ \psi_{k_1},\ \hat{w}_H^N, \hat{r}_1^N \rangle$ as the vector of relevant economic variables in (i) above and $a := \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$ as the vector of tax rates, parameters with respect to which we will perform comparative statics, and the remaining parameters that are going to be held fixed throughout the analysis.

3.1 Parametrising tax equilibria with respect to $\kappa,\ \beta,\ \psi_H,\ \psi_{k_1},\ \hat{w}_H^N,\ \hat{r}_1^N,\ \tau,\ \alpha_L,\ \alpha_1,$ and $\mathcal{P}$

In our numerical example, while solving for the socially optimal tax rates, we would like to study both the direct and indirect effects of changes in tax rates on the social welfare. In particular, we wish to delineate the indirect effects of changes in these tax rates on social welfare due to their effects on economic variables measuring migration, extent of education and skill formation by labour force migrating from the informal to the formal sector, long-run growth rates of physical and human capital in the formal sector, the net wage and rental rates in the formal sector. We will call all remaining effects of changes in tax rates on social welfare as the direct effects.

In the appendix, the macroeconomic tax equilibrium conditions for the general case that were derived in DM are rewritten for the special case of the pure formal sector that we focus on in this work. These take the form of five equations. We show in the appendix that they can be solved for the initial equilibrium levels of consumption of the formal sector good by the formal and informal households and the equilibrium levels of government infrastructure expenditures in the two sectors; in particular, they can be solved for $\mathcal{C}_{11}(0),\ \mathcal{C}_{12}(0),\ \mathcal{C}_1(0),\ G_1$, and G_2 as the following functions of $s := \langle \kappa,\ \beta,\ \psi_H,\ \psi_{k_1},\ \hat{w}_H^N, \hat{r}_1^N \rangle$ and $a := \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$.[8]

$$
\begin{aligned}
\mathcal{C}_{11}(0) &= \mathcal{C}_{110}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) = \mathcal{C}_{110}\left(s, a\right) \\
\mathcal{C}_{12}(0) &= \mathcal{C}_{120}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) = \mathcal{C}_{120}\left(s, a\right) \\
\mathcal{C}_{1}(0) &= \mathcal{C}_{10}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) = \mathcal{C}_{10}\left(s, a\right) \\
G_1 &= G_1\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) = G_1\left(s, a\right) \\
G_2 &= G_2\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) = G_2\left(s, a\right)
\end{aligned} \tag{4}
$$

[8] As seen in DM, initial consumptions of the formal sector good by the formal and informal households are increasing functions of $\mathcal{C}_{11}(0)$, and $\mathcal{C}_{12}(0)$, while $\mathcal{C}_1(0) = \mathcal{C}_{11}(0) + \mathcal{C}_{12}(0)$. The latter determines the equilibrium level of the public good available to households to consume.

3.2 Parametrising tax equilibria using τ, α_L, α_1, and $\mathcal{P}$

In our case, as shown below, κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ are themselves functions of $a = \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$:

In DM, β is obtained by the solving the following implicit function as a function $\beta = \hat{\beta}(\lambda_{hb}, \mathcal{P})$.

$$\frac{2S(\beta)}{S'(\beta)^2}\lambda_{hb}^2 - \left[\frac{\beta S'(\beta) - 2S(\beta)}{S'(\beta)}\right] r_b \lambda_{hb} + \left(r_b + w_2\right) = 0 \tag{5}$$

Noting from (2) that λ_{hb} is itself a function of $\langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle = a$, we obtain β as the following function of the function $\beta = \hat{\beta}(\lambda_{hb}(\tau, \alpha_L, \alpha_1, \mathcal{P}), \mathcal{P}) = \beta(\tau, \alpha_L, \alpha_1, \mathcal{P}) = \beta(a)$. We note from (1) and (3) that ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ are also functions of a. Further, as defined in DM, κ is also a function of a:

$$\begin{aligned} &\kappa = \hat{\kappa}(\lambda_{hb}, \beta) = \frac{2\lambda_{hb}}{S'(\beta)} = \kappa(\tau, \alpha_L, \alpha_1, \mathcal{P}) \;=\; \kappa(a), \quad \psi_{k_1}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \;=\; \psi_{k_1}(a), \\ &\psi_H(\tau, \alpha_L, \alpha_1, \mathcal{P}) = \psi_H(a), \quad \hat{w}_H^N(\tau, \alpha_L, \alpha_1, \mathcal{P}) \;=\; \hat{w}_H^N(a), \quad \hat{r}_1^N(\tau, \alpha_L, \alpha_1, \mathcal{P}) \;=\; \hat{r}_1^N(a) \end{aligned} \tag{6}$$

This implies that tax equilibrium values of $\mathcal{C}_{11}(0)$, $\mathcal{C}_{12}(0)$, $\mathcal{C}_1(0)$, G_1, and G_2 are eventually the following functions of $a := \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$:

$$\begin{aligned} \mathcal{C}_{1m}(0) &= \mathcal{C}_{1m0}\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\ &=: \hat{\mathcal{C}}_{1m0}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \equiv \hat{\mathcal{C}}_{1m0}(a) \;\forall\; m = 1, 2 \\ \mathcal{C}_1(0) &= \mathcal{C}_{10}\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\ &=: \hat{\mathcal{C}}_{10}(\tau, \alpha_L, \alpha_1, \mathcal{P}) \equiv \hat{\mathcal{C}}_{10}(a) \\ G_1 &= G_1\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\ &=: \hat{G}_1(\tau, \alpha_L, \alpha_1, \mathcal{P}) \equiv \hat{G}_1(a) \\ G_2 &= G_2\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\ &=: \hat{G}_2(\tau, \alpha_L, \alpha_1, \mathcal{P}) \equiv \hat{G}_2(a) \end{aligned} \tag{7}$$

Thus, provided it *exists*, a tax equilibrium can be computed for a given configuration of $\tau = \langle \tau_w, \tau_k, \tau_c \rangle \in [0,1] \times [0,1] \times [0,1]$ and for a given parameter vector $\langle \alpha_L, \alpha_1, \mathcal{P} \rangle$.

A necessary condition for existence of a tax equilibrium is that the shadow values of human capital H and the formal-sector physical capital K_1 relative to the shadow value of bonds B be real and non-negative, *i.e.*, $\lambda_{hb}(\tau_w, \mathcal{P}) \geq 0$ and $\lambda_{k_1 b}(\tau_k, \mathcal{P}) \geq 0$. From (2) it follows that the shadow prices are real if and only if[9]

$$(1 + r_b\theta)^2 - (1 + 2\theta(1 - \tau_w)\alpha_H) \geq 0 \tag{8}$$

$$(1 + \pi r_b)^2 - (1 + 2\pi(1 - \tau_k)\alpha_1) \geq 0 \tag{9}$$

It follows that there exist lower bars on τ_w and τ_k given by functions $\underline{\tau}_w(\alpha_L, \alpha_1, \mathcal{P})$ and $\underline{\tau}_k(\alpha_L, \alpha_1, \mathcal{P})$, respectively, such that for all $\tau_w < \underline{\tau}_w(\alpha_L, \alpha_1, \mathcal{P})$ or for all $\tau_k < \underline{\tau}_k(\alpha_L, \alpha_1, \mathcal{P})$ tax equilibria do not exist for the parameter vector $\langle \alpha_L, \alpha_1, \mathcal{P} \rangle$. These lower bars are given by

$$\underline{\tau}_i(\alpha_L, \alpha_1, \mathcal{P}) = \max\{\hat{\tau}_i, 0\} \quad \text{for} \quad i = w, k \tag{10}$$

where $\hat{\tau}_w$ and $\hat{\tau}_k$, respectively, satisfy (8) and (9) as equalities and are expressed totally in terms of the parameters.[10] For $i = w, k$, given values of the parameters, $\hat{\tau}_i$ is the minimum value that τ_i can take to ensure that the relative shadow price of the corresponding formal-sector capital resource is real and non-negative. For example, for a tax rate below $\hat{\tau}_k$, $\lambda_{k_1 b}$ does not take a real value; and for all values of tax rate above this level, $\lambda_{k_1 b}$ takes real and non-negative values. In general, the definition of $\hat{\tau}_i$ permits it to take negative values. To restrict analysis to the case where tax rates can take only non-negative values and the relative shadow price of the corresponding formal-sector capital resource is well defined, in (10), we define $\underline{\tau}_i$ as the maximum of zero and $\hat{\tau}_i$. Any tax rate τ_i that is greater than or equal to $\underline{\tau}_i$ will be non-negative and will result in the relative shadow price of the corresponding formal-sector capital resource taking non-negative real values.

The level of transfer made to the informal sector household at a tax equilibrium corresponding to $a := \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$ at the initial time point $t = 0$ is given by the function

$$\begin{aligned} T(0) &= \tau_b r_b B(0) + \tau_k r_1 K_1(0) + \tau_w w_1 H(0) + \tau_c C_1(0) + G_1(\mathcal{I}A_1 - 1) - G_c(0) - p(\mathcal{P})\, G_2 \\ &= \tau_b r_b B(0) + \tau_k r_1 K_1(0) + \tau_w w_1 H(0) + \tau_c \left[\hat{\mathcal{C}}_{10}(a) - 2\mu_1\right] + \hat{G}_1(a)(\mathcal{I}A_1 - 1) \\ &\quad - g_c \hat{\mathcal{C}}_{10}(a) - p(\mathcal{P})\, \hat{G}_2(a) = \hat{T}_0(a) \end{aligned} \tag{11}$$

[9]Note also from (2) that the two shadow prices are non-negative since θ and π are positive and τ_w and τ_k lie in the interval $[0, 1]$.

[10]That it, they are obtained by solving the following for τ_w and τ_k:

$$(1 + r_b\theta)^2 - (1 + 2\theta(1 - \tau_w)\alpha_H) = 0$$
$$(1 + \pi r_b)^2 - (1 + 2\pi(1 - \tau_k)\alpha_1) = 0$$

We will focus on an institutional structure where the value of transfer is restricted to take non-negative values. The very reasons that preclude taxation of the informal sector also preclude a negative transfer to (which is a positive lump-sum tax on) this sector. Thus, we will focus on tax equilibria where $\hat{T}_0(a) \geq 0$.

3.3 Tax equilibrium comparative statics.

Since the tax equilibrium values of κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ are functions of $a = \langle \tau, \alpha_L, \alpha_1, \mathcal{P} \rangle$, the remark below studies how equilibrium values of these economic variables will change as various tax rates and productivity levels of human and physical capital change. To sign the derivatives in Remark 1 below, we recall the functions λ_{hb} and $\lambda_{k_1 b}$ defined in (2) and the necessary conditions for the existence of a tax equilibrium, which follow from (8) and (9). In particular, if $\tau_w > \underline{\tau}_w(\alpha_L, \alpha_1, \mathcal{P})$ and $\tau_k > \underline{\tau}_k(\alpha_L, \alpha_1, \mathcal{P})$, then $(1 + r_b\theta)^2 - (1 + 2\theta(1 - \tau_w)\alpha_H) > 0$ and $(1 + \pi r_b)^2 - (1 + 2\pi\alpha_1(1 - \tau_k)) > 0$ and we have

$$\begin{aligned} \frac{\partial \lambda_{hb}}{\partial \tau_w} &= \frac{\partial \hat{\lambda}_{hb}}{\partial \hat{w}_H^N} \frac{\partial \hat{w}_H^N}{\partial \tau_w} = \frac{-\alpha_H \theta}{\sqrt{(1 + r_b\theta)^2 - (1 + 2\theta(1 - \tau_w)\alpha_H)}} < 0 \\ \frac{\partial \lambda_{k_1 b}}{\partial \tau_k} &= \frac{\partial \hat{\lambda}_{k_1 b}}{\partial \hat{r}_1^N} \frac{\partial \hat{r}_1^N}{\partial \tau_k} = \frac{-\alpha_1 \pi}{\sqrt{(1 + \pi r_b)^2 - (1 + 2\pi\alpha_1(1 - \tau_k))}} < 0 \end{aligned} \tag{12}$$

Remark 1 *The derivatives of κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ with respect to τ_w and τ_k are*

$$\begin{aligned} \frac{\partial \beta}{\partial \tau_w} &= \frac{\beta\left(-r_b\beta^2 + 4\lambda_{hb}\right)}{\lambda_{hb}\left[r_b\beta^2 + 2\lambda_{hb}\right]} \frac{\partial \lambda_{hb}}{\partial \tau_w} = \frac{\partial \hat{\beta}}{\partial \lambda_{hb}} \frac{\partial \lambda_{hb}}{\partial \tau_w} & \frac{\partial \beta}{\partial \tau_k} &= 0 \\ \frac{\partial \kappa}{\partial \tau_w} &= -\frac{4\lambda_{hb}}{\beta^3} \frac{\partial \beta}{\partial \tau_w} + \frac{2}{\beta^2} \frac{\partial \lambda_{hb}}{\partial \tau_w} = \frac{\partial \hat{\kappa}}{\partial \beta} \frac{\partial \beta}{\partial \tau_w} + \frac{\partial \hat{\kappa}}{\partial \lambda_{hb}} \frac{\partial \lambda_{hb}}{\partial \tau_w} & \frac{\partial \kappa}{\partial \tau_k} &= 0 \\ \frac{\partial \psi_H}{\partial \tau_w} &= \frac{1}{\theta} \frac{\partial \lambda_{hb}}{\partial \tau_w} = \frac{\partial \hat{\psi}_H}{\partial \lambda_{hb}} \frac{\partial \lambda_{hb}}{\partial \tau_w} < 0 & \frac{\partial \psi_H}{\partial \tau_k} &= 0 \\ \frac{\partial \psi_{k_1}}{\partial \tau_w} &= 0 & \frac{\partial \psi_{k_1}}{\partial \tau_k} &= \frac{1}{\pi} \frac{\partial \lambda_{k_1 b}}{\partial \tau_k} = \frac{\partial \hat{\psi}_{k_1}}{\partial \lambda_{k_1}} \frac{\partial \lambda_{k_1 b}}{\partial \tau_k} < 0 \\ \frac{\partial \hat{w}_H^N}{\partial \tau_w} &= -\alpha_H < 0 & \frac{\partial \hat{w}_H^N}{\partial \tau_k} &= 0 \\ \frac{\partial \hat{r}_1^N}{\partial \tau_w} &= 0 & \frac{\partial \hat{r}_1^N}{\partial \tau_k} &= -\alpha_1 < 0 \end{aligned} \tag{13}$$

The corresponding derivatives with respect to τ_c are all zero.

It follows from (13) that

- the negative signs of $\frac{\partial \hat{w}_H^N}{\partial \tau_w}$ and $\frac{\partial \hat{r}_1^N}{\partial \tau_k}$ reflect the reductions in net of tax returns per units of human and physical capital in the formal sector when the taxation rates of human capital (τ_w) or physical capital (τ_k) rise, respectively.

- the sign of $\frac{\partial \beta}{\partial \tau_w}$ is indeterminate. It is positive if $-r_b\beta^2 + 4\lambda_{hb} < 0$ and it is negative when $-r_b\beta^2 + 4\lambda_{hb} > 0$.

  Note that the amount of human capital contributed by every unit of unskilled labour leaving the informal sector and joining the formal sector after training is β. Thus, the amount of skilled labour/human capital supplied by X amount of unskilled labour force migrating to the formal is $\beta X = \beta\kappa L$.[11] Intuitively, this shows that there is some degree of substitutability between β and κ in the supply of effective units of skilled labour in the formal sector by the labour force migrating from the informal sector. Holding L fixed, the impact of a decrease in κ on the effective units of labour contributed by the migratory labour force can be offset by an increase in the level of skill factor β.

  The after-tax income received by the informal sector household from X amount of migration is

  $$\hat{w}_H^N \beta X = \hat{w}_H^N \beta\kappa L.$$

  Recall from (5) that β is a function of λ_{hb}, which in turn, as seen in (2), is a function of the net return to human capital, $\hat{w}_H^N$. Recalling the definition of $\hat{w}_H^N$ in (1), ceteris paribus, an increase in the taxation of skilled labour in the formal sector leads to a fall in $\hat{w}_H^N$ and hence tends to reduce the the income received by households from participation in production in this sector. In particular, ceteris-paribus, this negative impact on income from migration due to a rise in τ_w, can be mitigated by the informal sector household if it decides to increase the skill level of every unit of unskilled labour migrating to the formal sector. In that case, the derivative $\frac{\partial \beta}{\partial \tau_w}$ would take a positive value.

  Ceteris paribus, an increase in taxation of skilled labour reduces the income that the informal sector household is earning from its existing level of skilled labour supply to the formal sector. The derivative $\frac{\partial \beta}{\partial \tau_w}$ could be negative in this situation if this household is too poor to begin with, for then it may not be able to afford a the existing level of skill factor, β, and may seek to reduce the skill factor sought for its migrating labour to cut costs of education. This reflects a negative income effect on β of an increase in τ_w.

- The sign of $\frac{\partial \kappa}{\partial \tau_w}$ is indeterminate. It depends on the sign of $\frac{\partial \beta}{\partial \tau_w}$ and $\frac{\partial \lambda_{hb}}{\partial \tau_w}$.

  It follows from (6) that, holding λ_{hb} fixed, there is an inverse relationship between physical migration of unskilled labour κ and the skill factor sought by them, *i.e.*, $\frac{\partial \hat{\kappa}}{\partial \beta} < 0$.

[11] In DM, $\kappa = \frac{X}{L}$, is the share of migration in the net unskilled labour force employed in the formal sector.

Hence, if $\frac{\partial \beta}{\partial \tau_w}$ is positive (respectively, negative), then it has a decreasing (respectively, an increasing) effect on κ.[12]

Noting that an increase in shadow price of human capital has a positive effect on κ, *i.e.*, $\frac{\partial \hat{\kappa}}{\partial \lambda_{hb}} > 0$, we find that an increase in taxation of skilled labour, by reducing the relative shadow price of human capital, *i.e.*, $\frac{\partial \lambda_{hb}}{\partial \tau_w} < 0$, acts as a disincentive for migration of labour from the informal to the formal sector.

- Since long-run growth rates of physical and human capital in the formal sector, ψ_{k_1} and ψ_H, can also be interpreted as the levels of investments per unit stocks of human and physical capital, respectively, the signs of $\frac{\partial \psi_H}{\partial \tau_w}$ and $\frac{\partial \psi_{k_1}}{\partial \tau_k}$ are negative. This is because, ceteris paribus, an increase in τ_w or τ_k reduces the relative shadow prices of human and physical capital (λ_{hb} or λ_{hk_1}), respectively, disincentivising investment spending in the formal sector.[13]

Remark 2 below presents the tax equilibrium comparative statics with respect to the the productivities of the unskilled labour and the physical capital employed in the formal sector. Before presenting this remark, we first note that (1) and (2) imply:

$$\frac{\partial \lambda_{hb}}{\partial \alpha_L} = \frac{\partial \lambda_{hb}}{\partial \alpha_1} = \frac{\partial \lambda_{k_1 b}}{\partial \alpha_L} = 0 \quad \text{and} \quad \frac{\partial \lambda_{k_1 b}}{\partial \alpha_1} > 0 \tag{14}$$

Remark 2 *The partial derivatives of κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ with respect to α_L and α_1 are:*

$$\begin{array}{lclllcl}
\frac{\partial \beta}{\partial \alpha_L} & = & \frac{3 r_b \beta^2}{\alpha_2 \lambda_{hb}(2\lambda_{hb} + r_b \beta^2)} > 0 & & \frac{\partial \beta}{\partial \alpha_1} & = & 0 \\
\frac{\partial \kappa}{\partial \alpha_L} & = & \frac{-4\lambda_{hb}}{\beta^3}\frac{\partial \beta}{\partial \alpha_L} = \frac{\partial \hat{\kappa}}{\partial \beta}\frac{\partial \beta}{\partial \alpha_L} < 0 & & \frac{\partial \kappa}{\partial \alpha_1} & = & 0 \\
\frac{\partial \psi_H}{\partial \alpha_L} & = & 0 & & \frac{\partial \psi_H}{\partial \alpha_1} & = & 0 \\
\frac{\partial \psi_{k_1}}{\partial \alpha_L} & = & 0 & & \frac{\partial \psi_{k_1}}{\partial \alpha_1} & = & \frac{\partial \hat{\psi}_{k_1}}{\partial \lambda_{k_1 b}}\frac{\partial \lambda_{k_1 b}}{\partial \alpha_1} = \frac{1-\tau_k}{\sqrt{(1+\pi r_b)^2 - (1+2\pi(1-\tau_k)\alpha_1)}} > 0 \\
\frac{\partial \hat{w}_H^N}{\partial \alpha_L} & = & 0 & & \frac{\partial \hat{w}_H^N}{\partial \alpha_1} & = & 0 \\
\frac{\partial \hat{r}_1^N}{\partial \alpha_L} & = & 0 & & \frac{\partial \hat{r}_1^N}{\partial \alpha_1} & = & \alpha_1(1-\tau_k) > 0
\end{array} \tag{15}$$

As productivity of unskilled labour in the informal sector increases, ceteris paribus, there is an

[12] Recall that the effective units of labour supplied to the formal sector is $\beta \alpha_H \kappa L$. Thus, it is possible that, if an increase in τ_w stimulates an increase (respectively, a decrease) in β, then there is also a reduction (respectively, an increase) in κ in a manner that the total effective units of skilled labour supplied does not go down.

[13] Recall from (3) that the long run growth rates of human and formal-sector physical capital, ψ_H and ψ_{k_1}, are positively related to λ_{hb} and λ_{hk_1}, respectively.

increase in the skill factor β of the migrating labour force due to the positive income effect that an increase in α_L generates in this sector. People have more income in this sector and hence can afford to spend more on increasing the skills of the migrating labour force Hence, $\frac{\partial \beta}{\partial \alpha_L} > 0$.

Given the inverse relation between skill factor and migration that was discussed above and the positive impact that increase in α_L has on the skill factor sought by the informal sector for the migratory labour force, the impact of increase in α_L on migration is negative, *i.e.*, $\frac{\partial \kappa}{\partial \alpha_L} < 0$. If skill formation of the migrating labour force increases due to increase in productivity of the informal sector then the informal sector household can cut down on migration and save on unskilled labour force available for work in the informal sector, without affecting much its supply of effective units of skilled labour to the formal sector.

Ceteris-paribus, an increase in productivity of physical capital in the formal sector α_1 increases its shadow value $\lambda_{k_1 b}$ and hence incentivises greater investment per unit stock of formal sector physical capital. Hence, $\frac{\partial \psi_{k_1}}{\partial \alpha_1} > 0$. At the same time, it also increases its net return. Hence, $\frac{\partial \hat{r}_1^N}{\partial \alpha_1} > 0$.

4 Second-best social welfare maximisation and comparative statics of social welfare maximisation.

The social welfare at a tax equilibrium for a given configuration of tax rates and vector of parameters $a = \langle \tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P} \rangle$ is obtained from Section 4 in DS and from employing (7) as a function with image

$$\begin{aligned}
\mathbb{W} &= \bar{\mathcal{W}}\left(\mathcal{C}_{110}\left(s(a), a\right), \mathcal{C}_{120}\left(s(a), a\right), \mathcal{C}_{10}\left(s(a), a\right)\right) \\
&= \frac{1}{1-\varphi}\left[\left(\frac{1}{1-\epsilon}\right)\left(\frac{\eta_2(1+\tau_c)}{\eta_1 p}\right)^{\eta_2(1-\epsilon)}\left(g_c\ \mathcal{C}_{10}\left(s(a), a\right)\right)^{1-\epsilon}\left(\frac{1}{\rho - 2\psi_{\mathcal{C}}(1-\epsilon)}\right)\right]^{1-\varphi} \\
&\qquad \times \sum_{m=1}^{2} \mathcal{C}_{1m0}\left(s(a), a\right)^{(1-\epsilon)(1-\varphi)}
\end{aligned}$$

where $s(a) = \langle \kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a) \rangle$.

Social welfare can be redefined finally as a function of a as follows:

$$\begin{aligned}
\bar{\mathcal{W}}\left(\mathcal{C}_{110}\left(s(a), a\right), \mathcal{C}_{120}\left(s(a), a\right), \mathcal{C}_{10}\left(s(a), a\right)\right) &=: \mathcal{W}\left(s(a), a\right) \qquad (16) \\
&\equiv \mathcal{W}\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\
&=: \mathcal{W}^*(a) \qquad (17)
\end{aligned}$$

Under the maintained assumptions in DS, it follows from Section **??** that the social welfare maximisation is well defined only if the parameter vectors $\mathcal{P}$ lie in the set[14]

$$\Xi = \{\mathcal{P} \in \mathbf{R}_+^{15} \mid 2\psi_{\mathcal{C}}(1-\epsilon) - \rho < 0 \text{ and } n - r_b \leq 0\}$$

4.1 The social welfare maximisation problem and its first-order conditions.

Given $\mathcal{P} = \langle \rho, \epsilon, \varphi, \theta, \pi, A_1, A_2, \alpha_2, \alpha_H, \eta_2, \eta_1, \mu_1, \mu_2, r_b, n, g_c \rangle \in \Xi$ and values of α_L and α_1, consider the set of all tax equilibria consistent with parameters $\langle \alpha_L, \alpha_1, \mathcal{P} \rangle$. The socially optimal fiscal policy corresponding to parameters $\mathcal{P}, \alpha_L$, and α_1 is associated with the tax equilibrium in this set that maximises social welfare. Thus, it is obtained by solving the following social welfare maximisation problem:

$$\begin{aligned} \max_{\langle \tau_w, \tau_k, \tau_c \rangle \in [0,1]^3} \quad & \mathcal{W}^*(\tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P}) \equiv \mathcal{W}^*(a) = \mathcal{W}\left(\kappa(a), \beta(a), \psi_H(a), \psi_{k_1}(a), \hat{w}_H^N(a), \hat{r}_1^N(a), a\right) \\ & \text{subject to} \\ & (i) \;\; \tau_w \geq \hat{\tau}_w(\alpha_L, \alpha_1, \mathcal{P}), \quad (ii) \;\; \tau_k \geq \hat{\tau}_k(\alpha_L, \alpha_1, \mathcal{P}), \quad \text{and} \quad (iii) \;\; \hat{T}_0(a) \geq 0 \end{aligned} \tag{18}$$

where $a = \langle \tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P} \rangle$. Let the solution of this problem be given by[15]

$$\tau^* = \langle \tau_w^*, \tau_k^*, \tau_c^* \rangle = \langle \boldsymbol{\tau}_w(\alpha_L, \alpha_1, \mathcal{P}), \; \boldsymbol{\tau}_k(\alpha_L, \alpha_1, \mathcal{P}), \; \boldsymbol{\tau}_c(\alpha_L, \alpha_1, \mathcal{P}) \rangle =: \boldsymbol{\tau}(\alpha_L, \alpha_1, \mathcal{P})$$

and let the socially optimal level of the initial transfer to the informal sector be denoted by[16]

$$T_0^* = \hat{T}_0(\tau^*, \alpha_L, \alpha_1, \mathcal{P}) = \mathbf{T}_0^*(\alpha_L, \alpha_1, \mathcal{P})$$

Let $\xi_{\hat{\tau}_w}$, $\xi_{\hat{\tau}_k}$, and ξ_T be the Lagrange multipliers of constraints (i), (ii), and (iii) of problem (18), respectively. Recalling (17), the Kuhn-Tucker first order conditions of social welfare

[14]These conditions follow from constraints (iv) and (vii) of social welfare maximisation in Section 7 in DS with $\tau_b = 0$, which is assumed in the special case of pure formal sector that is being studied here.

[15]Note that at the solution $\tau_i^* \geq \underline{\tau}_i(\alpha_L, \alpha_1, \mathcal{P})$ for $i = w, k$, where $\underline{\tau}_i(\alpha_L, \alpha_1, \mathcal{P})$ is defined in (10).

[16]This follows from (11).

maximisation that also allow for corner solutions are

$$\frac{\partial \mathcal{W}^*}{\partial \tau_i} \equiv$$
$$\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{w}_L^N}\frac{\partial \hat{w}_L^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \tau_i}$$
$$+\xi_T \frac{\partial \hat{T}_0}{\partial \tau_i} + \xi_{\hat{\tau}_i} \leq 0, \quad \tau_i^* \geq 0, \quad \tau_i^* \frac{\partial \mathcal{W}^*}{\partial \tau_i} = 0;$$
$$\hat{\tau}_i\left(\alpha_L, \alpha_1, \mathcal{P}\right) - \tau_i^* \leq 0, \quad \xi_{\hat{\tau}_i} \geq 0, \quad \left[\hat{\tau}_i\left(\alpha_L, \alpha_1, \mathcal{P}\right) - \tau_i^*\right]\xi_{\hat{\tau}_i} = 0; \quad \forall\ i = w, k \tag{19}$$

$$\frac{\partial \mathcal{W}^*}{\partial \tau_c} \equiv$$
$$\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \hat{w}_L^N}\frac{\partial \hat{w}_L^N}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_c} + \frac{\partial \mathcal{W}}{\partial \tau_c}$$
$$+\xi_T \frac{\partial \hat{T}_0}{\partial \tau_c} \leq 0, \quad \tau_c^* \geq 0, \quad \text{and} \quad \tau_c^* \frac{\partial \mathcal{W}^*}{\partial \tau_c} = 0; \tag{20}$$

$$\hat{T}_0\left(\tau^*, \alpha_L, \alpha_1, \mathcal{P}\right) \geq 0, \quad \xi_T \geq 0, \quad \hat{T}_0\left(\tau^*, \alpha_L, \alpha_1, \mathcal{P}\right)\xi_T = 0 \tag{21}$$

4.2 Some solution types of social welfare maximisation

The remark below outlines three types of solutions to the social welfare maximisation problem (18) that we find in our numerical example. As we will find in the context of this numerical example, some of the tax rates can well be zero at a social optimum depending upon the parameter values adopted; hence the relevance of specifying Kuhn-Tucker conditions that allow for both corner and interior solutions to the social welfare maximisation problem. The intuition and interpretation of the different solution types reported in the remark below will be discussed in detail later in this work when we present our numerical example.

Remark 3 *In our numerical example, we encounter the following three types of solutions of the social welfare maximisation problem (18). All these solutions have the common feature that constraints (i) and (ii) of the problem are non-binding at the optimal configuration of tax rates* τ^* *so that the Lagrange multipliers* $\xi_{\hat{\tau}_w}$ *and* $\xi_{\hat{\tau}_k}$ *are both zero:*

1. *Constraint (iii) of problem (18) is non-binding at the optimum, so that the the socially optimal transfer is positive, i.e.,* $T_0^* > 0$*, and the Lagrange multiplier on constraint (iii) is zero, i.e.,* $\xi_T = 0$*. If there is an interior solution* τ_i *for some* $i = w, k, c$*, i.e.,* $\tau_i^* > 0$*,*

then it follows from the Kuhn-Tucker conditions that

$$\frac{\partial \mathcal{W}^*}{\partial \tau_i} \equiv \tag{22}$$
$$\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{w}_L^N}\frac{\partial \hat{w}_L^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \tau_i} = 0$$

2. *Constraint (iii) of problem (18) is non-binding at the optimum, so that the the socially optimal transfer is positive, i.e.,* $T_0^* > 0$*, and the Lagrange multiplier on constraint (iii) is zero, i.e.,* $\xi_T = 0$*. Suppose that for some* $i = w, k, c$ *the following is true:*

$$\frac{\partial \mathcal{W}^*}{\partial \tau_i} \equiv \tag{23}$$
$$\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{w}_L^N}\frac{\partial \hat{w}_L^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \tau_i} < 0,$$

Then the Kuhn-Tucker conditions imply that $\tau_i^* = 0$*, i.e., there is a corner solution for* τ_i.

3. *Constraint (iii) of problem (18) is binding at the optimum, so that the Lagrange multiplier on that constraint is positive, i.e.,* $\xi_T > 0$*, and the Kuhn-Tucker condition (21) implies that* $T_0^* = 0$ *at the optimum. At an interior solution for* τ_i *for some* $i = w, k, c$ *with* $\frac{\partial \hat{T}_0}{\partial \tau_i} > 0$*, conditions (19) and (20) imply that*

$$\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{w}_L^N}\frac{\partial \hat{w}_L^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_i} + \frac{\partial \mathcal{W}}{\partial \tau_i}$$
$$= -\xi_T \frac{\partial \hat{T}_0}{\partial \tau_i} < 0.$$

4.3 Direct and indirect effects of changing tax rates on social welfare.

The first order conditions (19) and (20) with respect to any tax rate τ_i involve decomposing the effect of changes in τ_i on social welfare into *direct* and *indirect* effects. Thus, for example, $\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_i}$ captures the indirect effect that a change in τ_i has on social welfare by first affecting κ, the migration per-unit of net labour force in the informal sector denoted by. This in turn is decomposed into the effect of migration on social welfare $\frac{\partial \mathcal{W}}{\partial \kappa}$ and the effect of change in i^{th} tax rate on migration $\frac{\partial \kappa}{\partial \tau_i}$.

Similarly too, we can define the indirect effects that a change in τ_i has on social welfare by affecting choice of average skill/education levels sought by the migrating labour force β, investment per unit stock of human capital ψ_H, investment per-unit of physical capital stock in the formal sector ψ_{k_1}, and net returns to human and physical capital in the formal sector $\hat{w}_H^N$ and $\hat{r}_1^N$, respectively. Recall that $\psi_H = \gamma + n$ is also the long-run rate of growth of human capital in the formal sector if $\gamma > 0$ and ψ_{k_1} is also the long-run rate of growth of the physical capital in the formal sector.

The direct effect of a change in τ_i on social welfare captures all the remaining effects that a change in τ_i has on social welfare.

We intuitively discuss below some of the channels through which κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ can affect social welfare in our model by affecting income and investment expenditures and hence consumption levels of the two households in our model. This is because, social welfare is a function of individual utilities, which are in turn functions of the consumption levels of the two representative households.

- *Effect of migration on social welfare – the sign of* $\frac{\partial \mathcal{W}}{\partial \kappa}$:

  There are both welfare costs and benefits of changes in κ, the migration per-unit of net labour force in the informal sector. The welfare costs include the costs such as the loss in output/income levels in the informal sector as a result of migration/(*i.e.,* loss in labour force) from this sector and the monetary cost incurred by migrating labour force on educating itself so as to be able to contribute to the skilled labour force in the formal sector. The welfare benefits of migration include the gain in output/income in the formal sector due to increase in human capital/skilled labour through migration into this sector as well as the resulting increase in tax revenue to the government due to additional income generated in this taxable sector because of migration. This increase in tax revenue can be used to increase transfer income to the informal sector and also to finance the public good and public infrastructural activities.

  A-priori, these costs and benefits make the sign of the net social marginal benefit from migration given by $\frac{\partial \mathcal{W}}{\partial \kappa}$ ambiguous.

- *Effect of choice of educational/skill levels sought by migrating labour force on social welfare – the sign of* $\frac{\partial \mathcal{W}}{\partial \beta}$:

  Ceteris paribus, an increase in β increases the additions to the stock of human capital by the migrating labour force and hence the income of the informal sector household for the same amount of migration. On the other hand, it also increases the cost of education/skill formation that the informal sector household has to incur on its migratory labour force.

Once again, the presence of both costs and benefits of increasing β make the sign of the net social marginal benefit from increasing the average skill factor of the migrating labour force given by $\frac{\partial \mathcal{W}}{\partial \beta}$ a-priori ambiguous.

- *Effects of changes in the amount of formal sector investments per-unit of human and physical capital stocks on social welfare– the signs of* $\frac{\partial \mathcal{W}}{\partial \psi_H}$ *and* $\frac{\partial \mathcal{W}}{\partial \psi_{k_1}}$:

  Given limited budgets, increases in the investment levels per-unit of human or physical capital in the formal sector given by ψ_H or ψ_{k_1}, respectively, will firstly, ceteris paribus, lead to greater diversion of spending of incomes of the households towards investment and the concomitant adjustment costs leading to lower consumption expenditures and hence lower social welfare. On the other hand, these will also increase the human or physical capital stocks in the formal sector and hence increase household incomes from these resources in the future. This can, in turn, potentially increase future consumption expenditures and hence individual and social intertemporal welfare. Hence, the signs of $\frac{\partial \mathcal{W}}{\partial \psi_H}$ and $\frac{\partial \mathcal{W}}{\partial \psi_{k_1}}$ are a-priori indeterminate.

- *Effects of changes in the net returns to human or physical capital on social welfare– the signs of* $\frac{\partial \mathcal{W}}{\partial \hat{w}_H^N}$ *and* $\frac{\partial \mathcal{W}}{\partial \hat{r}_1^N}$:

  Increases in net returns to formal sector physical or human capital given by $\hat{r}_1^N$ or $\hat{w}_H^N$, respectively, increase social welfare by increasing incomes of the two households. Thus, one expects that $\frac{\partial \mathcal{W}}{\partial \hat{w}_H^N} \geq 0$ and $\frac{\partial \mathcal{W}}{\partial \hat{w}_H^N} \geq 0$

- *Direct/residual effects of changes in tax rates on social welfare– the signs of* $\frac{\partial \mathcal{W}}{\partial \tau_i}$*, for* $i = w, k, c$:

  The direct/residual effects of changes in tax rates on social welfare include all effects of changes in these rates on social welfare other than those that are routed through levels of migration, choice of skill factor, investment per unit of human and physical capital resources, and the net returns to human and physical capital. These include the direct impact of changes in these rates on tax revenue of the government, which impacts social welfare by impacting the transfer to the informal sector and other public expenditures it finances, and are captured by the partial derivative $\frac{\partial \mathcal{W}}{\partial \tau_i}$. It also includes the reductions in the costs of investment (including adjustment costs) in human and physical capital induced by such tax increases as these costs are positively related to their respective shadow prices, which will fall when there are increases in taxation of human or physical capital, respectively.[17]

The effects of changes in κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ on social welfare have to be multiplied by the effects that changes in tax rates have on κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$, respectively, to obtain the indirect effects of changes in tax rates on social welfare. The effects of changes in tax rates on κ, β, ψ_H, ψ_{k_1}, $\hat{w}_H^N$, and $\hat{r}_1^N$ have been tabulated in (13).

4.4 Comparative statics of social welfare maximisation.

The shadow prices of human and physical capital employed in the formal sector and the socially optimal levels of migration, average skill factor, net returns from physical and human capital, and the share of investments in physical and human capital are obtained as functions of all the parameters using (2), (5), and (6).

$$\begin{aligned} \mathcal{Q} &= \mathcal{Q}^*(\alpha_L, \alpha_1, \mathcal{P}) := \mathcal{Q}(\boldsymbol{\tau}(\alpha_L, \alpha_1, \mathcal{P}), \alpha_L, \alpha_1, \mathcal{P}) \\ & \text{for} \quad \mathcal{Q} = \lambda_{hb},\ \lambda_{k_1 b},\ \kappa,\ \beta,\ \psi_{k_1},\ \psi_H,\ \hat{w}_H^N,\ \hat{r}_1^N \end{aligned} \tag{24}$$

In what follows, we will use these functions to study how the socially optimal levels of migration, average skill factor, net returns from physical and human capital, and the share of investments in physical and human capital of skill factor change due to changes in the productivities of unskilled labour in the informal sector and the physical capital in the formal sector.

5 Numerical simulations.

5.1 Base parameters.

The base values of the parameters used to solve the social welfare maximisation problem to obtain the optimal fiscal policy and related socially optimum indicator values along with their description are listed in Tables 1 and 2. Table 1 provides the base values of the initial endowments of the representative households in the formal and informal sectors. Table 2 provides the base values of the rest of the parameters involved in the model. The formal sector household

[17]Recall, for example, that the cost of investment in human capital is $I_{hm}\left(1 + \frac{\theta I_{hm}}{2H}\right) = H_m \frac{I_{hm}}{H_m}\left(1 + \frac{\theta I_{hm}}{2H_m}\right) = H_m\left(\frac{\lambda_{hb}^2 - 1}{\theta}\right) = \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)$.

is well off compared to the informal sector household in the beginning in terms of its initial endowment of physical capital and human capital. We assume that the formal sector household owns 70% of the initial endowment of human capital. Since in this chapter we are focusing on the pure formal sector case of the general model in Chapter 3, the entire amount of the initial endowments of the unskilled labour force and the informal-sector physical capital are owned by the informal sector household. The representative formal sector household owns 90% of the initial endowment of the formal-sector physical capital. We also assume that the initial bond endowment of the formal sector household is 90% of the aggregate stock of bonds in the economy.

Table 1: Initial wealth differential between formal and informal sector

Initial endowments	Description	Base values
$l(0)$	total unskilled labour force at $t = 0$	100000000
$l_1(0)$	total unskilled labour force in formal sector at $t = 0$	0
$l_2(0)$	total unskilled labour force in informal sector at $t = 0$	100000000
$L(0)$	aggregate economy-wide net unskilled labour force at $t = 0$	10000000
$L_1(0)$	net unskilled labour force in formal sector at $t = 0$	0
$L_2(0)$	net unskilled labour force in informal sector at $t = 0$	10000000
$H(0)$	aggregate economy-wide human capital stock at $t = 0$	1000000000
$H_1(0)$	stock of human capital hold by the representative formal sector household at $t = 0$	700000000
$H_2(0)$	stock of human capital hold by the representative informal sector household at $t = 0$	300000000
$K_1(0)$	stock of physical capital in formal sector at $t = 0$	100000000
$K_2(0)$	stock of physical capital in informal sector at $t = 0$	10
$B(0)$	stock of foreign bond at $t = 0$	1000000
$B_1(0)$	foreign bonds holding by representative household in formal sector at $t = 0$	900000
$B_2(0)$	foreign bonds holding by representative household in informal sector at $t = 0$	100000
$K_{11}(0)$	stock of physical capital in formal sector hold by representative household in formal sector at $t = 0$	90000000
$K_{12}(0)$	stock of physical capital in formal sector hold by representative household in informal sector at $t = 0$	10000000
$K_{21}(0)$	stock of physical capital in informal sector hold by representative household in formal sector at $t = 0$	0
$K_{22}(0)$	stock of physical capital in informal sector hold by representative household in informal sector at $t = 0$	10

We measure human capital in effective units of skilled labour, which is defined as the physical units of skilled labour (*e.g.* labour time) multiplied by its marginal productivity in producing the formal-sector good, *i.e.,* good 1.[18] Thus, the unit of measurement, marginal productivity,

[18]The concept of effective units of labour is often employed in the public economics literature, see *e.g.,* Mirrlees

and price of the effective unit of skilled labour are the same as that of good 1. Hence, $w_1 = \alpha_H = 1$. The extent of effective units of skilled labour supplied by X units of migrating unskilled labour after acquiring skill factor β is $X\beta$.[19]

Our numerical example performs comparative static analyses for measuring the impact of changes in the productivities of formal-sector physical capital and unskilled labour force in the informal sector on optimal fiscal policies. For this purpose we simulate the model for a different values of α_L and α_1. The sets of values considered are provided in Table 2.

We assume that the degree of adjustment cost per unit investment in human capital, θ, is higher than that for physical capital, π, because an increase in human capital involves a learning process through acquiring education, training for skill development, etc., that is quite costly in terms or resources. We assume that the sector-specific provision of infrastructure expenditure by the government enhances the output level in each sector and the formal sector's output is more responsive to the provision of government infrastructure expenditure. So we assign a higher base value to the marginal productivity of the government's infrastructural expenditures in the formal sector than that in the informal sector: $A_1 > A_2$. Each representative household spends same proportion, $\eta_1 = \eta_2 = 0.5$, of their supernumerary income after paying for their subsistence consumption of informal sector's good. Assigning $\varphi = 0$, we consider a Benthamite/utilitarian social welfare function where we treat the representative households' welfare equally irrespective of their endowments or wealth.[20] Population grows at the rate of 1.5% annually. The base parameter values along with the assumption that $\tau_b = 0$ fix the price of good 2 as $p = 0.764006791$.

Recall that the skill formation costs for the migratory labour are assumed to take the form $X_m\left(1 + \frac{S(\beta_m)X_m}{2L_m}\right)$ whenever $X_m > 0$. Function S is assumed to be increasing and strictly convex and has to satisfy the condition $S'(\beta)\beta > 2S(\beta)$. In our numerical exercise, we assume that function S takes the form $S(\beta) = \frac{1}{3}\beta^3$. It can be verified that this form for S has the required properties.

(1971) and Atkinson and Stiglitz (1976, 1980). The amount of effective units of labour supplied by one physical unit of labour is equal to the marginal productivity of the physical unit of labour.

[19]Of course, acquisition of skill factor β by the unskilled migratory labour force is costly.

[20]While the social marginal utility is the same for both households under a utilitarian social welfare function, it does not follow that the social marginal utilities of consumption of the two households are also equal under this social welfare function.

Table 2: The base parameter values

Parameters	Description	Base values
$\mathcal{M}$	number of representative households in the economy	2
$\mathcal{J}$	number of firms in the formal sector	1
r_b	rate of return on foreign bond holding	0.09
α_1	marginal productivity of physical capital in formal sector	{0.119, 0.15, 0.18, 0.2, 0.25, 0.28}
α_2	marginal productivity of physical capital in informal sector	0.1178
α_H	marginal productivity of human capital in formal sector	1
α_L	marginal productivity of unskilled labour in informal sector	{38,40,45,47,48}
π	degree of adjustment cost for per unit physical capital investment in formal sector	10
θ	degree of adjustment cost for per unit human capital investment in formal sector	250
δ_1	depreciation rate of physical capital in formal sector	0
δ_2	depreciation rate of physical capital in informal sector	0
A_2	marginal productivity of government's infrastructural expenditures in informal sector	0.5
A_1	marginal productivity of government's infrastructural expenditures in formal sector	10
g_c	government's spending on public good share to formal sector's consumption net of its subsistence level consumption	0.0001
ρ	rate of time preference of consumer	0.064
ϵ	reciprocal of elasticity of substitution between consumption of formal and informal sectors' goods	0.9
η_1	proportion of supernumerary income spent on consumption of formal sector's good	0.5
η_2	proportion of supernumerary income spent on consumption of informal sector's good	0.5
μ_1	minimum consumption of formal sector' good after subsistence level consumption of informal sector's good	$1.88134 * 10^5$
μ_2	subsistence level consumption of informal sector's good	$1.84976 * 10^5$
φ	reciprocal of elasticity of substitution between the utility levels of the representative households in formal and informal sectors	0
n	population growth rate	0.015

5.2 Computation methodology.

We create a code in the computer software MATLAB that can, given the base vector of parameter values $\mathcal{P}$, a combination of productivities of the unskilled labour force and the formal-sector physical capital, $\langle \alpha_L, \alpha_1 \rangle$, and the initial distributions of endowments in the economy, compute the set of associated tax equilibria. This is done by specifying, for each of the three tax rates, τ_w, τ_k, and τ_c, a grid of ℓ equally-spaced values in the interval $[0, 1]$ that it can take.[21] Assuming the same grid for each tax rate, denoted by $\mathcal{L}$, the grid can be specified to be as fine as desired by the researcher by making ℓ sufficiently bigger, where ℓ is a non-negative integer. The set of all possible configurations of tax rates $\langle \tau_w, \tau_k, \tau_c \rangle$ made possible by our specifications of the grids is given by $\mathcal{L}^3 := \mathcal{L} \times \mathcal{L} \times \mathcal{L}$.

For the base values of parameters and endowment distributions and for a given vector of productivities $\langle \alpha_L, \alpha_1 \rangle$, our code computes the tax equilibrium, if it exists, for any given configuration of tax rates $\langle \tau_w, \tau_k, \tau_c \rangle \in \mathcal{L}^3$. To do so, it solves the system of five tax equilibrium equations (27) to (31) for the five unknowns $\mathcal{C}_{11}(0)$, $\mathcal{C}_{12}(0)$, $\mathcal{C}_1(0)$, G_1, and G_2. These are evaluated at the base parameter values, the initial levels of endowments, and the specified levels of productivities $\langle \alpha_L, \alpha_1 \rangle$. Doing so for every $\langle \tau_w, \tau_k, \tau_c \rangle \in \mathcal{L}^3$ generates the set of all possible tax equilibria associated with all the configurations of tax rates made possible by our grid given values of parameters, productivities $\langle \alpha_L, \alpha_1 \rangle$, and endowment distributions.

The MATLAB programme code then searches for that tax equilibrium in this set that leads to highest social welfare.

This provides a first estimate of the optimal fiscal policy corresponding to the base value of parameters and endowment distributions and for a given vector of productivities $\langle \alpha_L, \alpha_1 \rangle$. These estimates are revised and made finer by employing the software Mathematica. For example, when socially optimal tax rates computed by MATLAB are zero for two of the three tax rates and social welfare is strictly concave and has a maximum with respect to the remaining (third) tax rate, then this software allows us in plotting the graph of the net social marginal benefit function for the remaining tax rate and in computing its root (when it exists). Such a root can be interpreted as the socially optimal value of the third tax rate. This way the estimate for the optimal value of the third tax rate obtained from MATLAB, which lies in the relatively coarser grid $\mathcal{L}$, is refined further. On the other hand, even when social welfare is decreasing in some tax rates and hence does not have a stationary maximum value, Mathematica allows us

[21] Recall, in the special case of the general model in Chapter 3, which is being studied in this work, we have precluded taxation of returns from bond holding, *i.e.*, $\tau_b = 0$.

to compute the values of $\hat{\tau}_i$ and $\underline{\tau}_i$ as defined in (10), which can be used to revise the estimates of optimal tax rates obtained from MATLAB.

6 Results from the numerical simulations and interpretations.

The main comparative static results obtained from this numerical example are displayed in Tables 3 and 6. Table 3 presents the optimal values that fiscal policy variables and other economic variables of interest take during a comparative static study of social welfare maximisation where the productivity differential between labour in the formal and informal sectors changes; in particular five different values of α_L, the marginal product of unskilled labour in the informal sector, are considered, while holding the value of the marginal product of skilled labour/human capital in the formal, α_H, fixed at the base level. Similarly, Table 6 presents the optimal values that fiscal policy variables and other economic variables take when we do a comparative static study of social welfare maximisation by varying the productivity of the physical capital in the formal sector, α_1. Six different values of α_1 are considered, while holding the productivity of physical capital in the informal sector, α_2, fixed at the base level.

6.1 Comparative statics with respect to changes in the productivity of informal sector labour force, α_L.

6.1.1 Impact of changes in α_L on socially optimal levels of migration, skill factor of migratory labour force, net rate of returns and long-run growth rates of human and physical capital.

Table 3 indicates that there is no taxation of physical capital or consumption in the formal sector at the social optimum ($\tau_k^* = \tau_c^* = 0$), while the tax rate on returns from human capital/skilled labour, τ_w^*, decreases from 19% to 8% as the productivity of unskilled labour in the informal sector increases, that is, as the labour productivity differential between the two sectors decreases with the value of α_L increasing from 38 to 48. We study below the impact of these changes in the optimal fiscal policy on migration, skill factor of migratory labour force, net rate of returns and long-run growth rates of human and physical capital.

Average skill factor of the migratory labour force at the social optimum, β^*, increases from

Table 3: Impact of changes in productivity of informal sector labour force on optimal fiscal policy and other economic variables

	Values of α_L				
	38	**40**	**45**	**47**	**48**
τ_w^*	0.190925	0.167285	0.112859	0.0929236	0.0833433
τ_k^*	0	0	0	0	0
τ_c^*	0	0	0	0	0
T_0^*	1052200	33340200	122695000	162193000	182819000
G_1^*	135950000	149777000	184804000	199035000	206205000
G_2^*	1849890000	1939550000	2163970000	2253800000	2298730000
κ^*	0.00293312	0.00300588	0.00318562	0.00325654	0.00329178
β^*	88.118	88.9286	90.7689	91.4422	91.7673
γ^*	0.02655	0.0285427	0.0334925	0.0354603	0.0364417
$\hat{w}_H^{N^*}$	0.809075	0.832715	0.887141	0.907076	0.916657
$\hat{r}_1^{N*}$	0.119	0.119	0.119	0.119	0.119
$\psi_{k_1}^*$	0.042	0.042	0.042	0.042	0.042
ψ_H^*	0.042	0.044	0.048	0.050	0.051
$\mathcal{C}_{11}(0)$	351865000	367085000	404892000	419922000	427418000
$\mathcal{C}_{12}(0)$	644834000	679145000	765266000	799835000	817142000
$\mathcal{C}_1(0)$	996699000	1046230000	1170158000	1219757000	1244560000
U_1^*	3937.77	3973.71	4058.03	4089.79	4105.28
U_2^*	4183.67	4225.86	4324.76	4361.98	4380.14
$U_2^* - U_1^*$	245.9	252.15	266.73	272.19	274.86
$U^* = W^*$	8121.44	8199.57	8382.79	8451.77	8485.42
y_1^*	1010690000	1010690000	1010690000	1010690000	1010690000
y_2^*	291622000	306902000	345102000	360382000	368022000
$y_1^* - y_2^*$	719068000	703788000	665588000	650308000	642668000
$\psi_{\mathcal{C}}^*$	0.029	0.029	0.029	0.029	0.029
$\psi_{k_2}^*$	0.029	0.029	0.029	0.029	0.029
ψ_B	0.042	0.044	0.048	0.050	0.052
ψ_{Y_1}	0.042	0.044	0.048	0.050	0.052
ψ_{Y_2}	0.029	0.029	0.029	0.029	0.029
$\hat{X}^*$	0.00240	0.00252	0.00262	0.00267	0.00271
λ_{hb}^*	11.388	11.886	13.123	13.615	13.860
$\lambda_{k_1}^*$	1.420	1.420	1.420	1.420	1.420
p^*	0.764	0.764	0.764	0.764	0.764
$\hat{\tau}_w$	-0.1025	-0.1025	-0.1025	-0.1025	-0.1025
$\hat{\tau}_k$	-0.0966	-0.0966	-0.0966	-0.0966	-0.0966
$\underline{\tau}_w$	0	0	0	0	0
$\underline{\tau}_k$	0	0	0	0	0

Note:1. y_1^* and y_2^* represent initial income in formal and informal sectors respectively
2. numerical simulation are based on parameter values: $\alpha_1 = 0.119, \alpha_2 = 0.1178$

88.118 to 91.768 as α_L increases from 38 to 48. Hence, in this numerical example, we have

$$\frac{\partial \beta^*}{\partial \alpha_L} = \frac{\partial \beta}{\partial \tau_w}\frac{\partial \boldsymbol{\tau}_w}{\partial \alpha_L} + \frac{\partial \beta}{\partial \alpha_L} > 0$$

where we have employed (24). Here $\frac{\partial \beta}{\partial \tau_w}$ and $\frac{\partial \beta}{\partial \alpha_L}$ are obtained from (13) and (15), respectively. As seen, $\frac{\partial \beta^*}{\partial \alpha_L}$ can be decomposed into (i) $\frac{\partial \beta}{\partial \tau_w}\frac{\partial \tau_w^*}{\partial \alpha_L}$ – this is the indirect effect that a change in α_L has on β by first influencing the optimal fiscal policy (in particular, τ_w) (ii) $\frac{\partial \beta}{\partial \alpha_L}$ – this is the direct effect that a change in α_L has on β because of the income effects it induces in the informal sector. The former change (i) is negative because Panel 2 of Table 4, which tabulates first derivatives of various economic variables for $\alpha_L = 38$, shows that $\frac{\partial \beta}{\partial \tau_w}$ is positive and Table 3 shows that the optimal tax on human capital/skilled labour decreases with increase in α_L (*i.e.*, $\frac{\partial \tau_w^*}{\partial \alpha_L} < 0$).[22] Thus, on the one hand, an increase in the productivity of unskilled labour in the informal sector has a decreasing effect on the socially optimal level of skill factor as it implies a decrease in the optimal taxation of formal sector labour force, which in turn increases the net returns to human capital in the formal sector $\hat{w}_H^N$ and has a disincentive/*relaxing* effect on the level of skill factor sought by the migrating informal sector labour force.[23] On the other hand, an increase in productivity of the informal sector unskilled labour force has a direct positive income effect on β by increasing income in the informal sector, making acquisition of a higher skill factor more affordable for the migrating labour force of this sector. Panel 2 of Table 4 shows that $\frac{\partial \beta}{\partial \alpha_L}$ is positive. In the case of this numerical example, the net effect of a change in α_L on the socially optimal skill factor of migratory labour force is positive: $\frac{\partial \beta^*}{\partial \alpha_L} > 0$, *i.e.* the positive income effect of an increase in α_L offsets the the negative effect due to a decrease in τ_w^* that it induces.

Table 3 also shows that κ^*, which is defined as migration per unit of net labour-force employed in the informal sector, increases as α_L increases. Recalling from (6) that κ is a function of β and λ_{hb}, this implies that

$$\frac{\partial \kappa^*}{\partial \alpha_L} = \frac{\partial \hat{\kappa}}{\partial \beta}\frac{\partial \beta^*}{\partial \alpha_L} + \frac{\partial \hat{\kappa}}{\partial \lambda_{hb}}\frac{\partial \lambda_{hb}^*}{\partial \alpha_L} = \frac{\partial \hat{\kappa}}{\partial \beta}\frac{\partial \beta^*}{\partial \alpha_L} + \frac{\partial \hat{\kappa}}{\partial \lambda_{hb}}\left[\frac{\partial \lambda_{hb}}{\partial \tau_w}\frac{\partial \tau_w^*}{\partial \alpha_L} + \frac{\partial \lambda_{hb}}{\partial \alpha_L}\right] > 0$$

It follows from (6) that, holding λ_{hb} fixed, there is an inverse relationship between physical

[22] Panels 1 and 2 of Table 4 tabulate the derivatives of their column variables with respect to their row variables. Thus, the $(1,3)^{\text{th}}$ entry of the matrix in Panel 1 is $\frac{\partial W}{\partial \beta}$, while that of the matrix in Panel 2 is $\frac{\partial \psi_H}{\partial \tau_w}$. Qualitatively similar results will also be obtained when Table 4 is computed for values of α_L other than $\alpha_L = 38$.

[23] There is an incentive for migratory labour force to increase skill factor to compensate for fall in net return to human capital. However, in the case under study, the net return to human capital increases due to fall in τ_w^*. This implies that they can now go in for lower skill factor development – spend less on acquiring skill factor.

Table 4: First derivatives of relevant economic variables

Panel 1

	u_1	u_2	$\mathcal{W} = u_1 + u_2$	$\mathcal{C}_{110}$	$\mathcal{C}_{120}$	$\mathcal{C}_{10}$
β	0.048309	0.130658	0.178967	0	122276	122276
κ	-2869.41	-7760.72	-10630.1	0	$-7.26284 * 10^9$	$-7.26284 * 10^9$
ψ_H	3569.6	-2383.75	1185.85	$3.27518 * 10^9$	$-3.5174 * 10^9$	$-2.4222 * 10^8$
ψ_{K_1}	13.4894	-7.82039	5.66904	$1.20537 * 10^7$	$1.20537 * 10^7$	$4.84288 * 10^-8$
$\hat{w}_H^{N*}$	1545.3	1861.66	3406.96	$6.30599 * 10^8$	$1.49451 * 10^9$	$2.12511 * 10^9$
$\hat{r}_1^{N*}$	138.314	163.324	301.638	$5.73415 * 10^7$	$1.30322 * 10^8$	$1.87663 * 10^8$
τ_w	1164.49	2155.66	3320.15	$2.70397 * 10^8$	$1.91115 * 10^9$	$2.18155 * 10^9$
τ_k	12.1701	21.9223	34.0924	$2.9909 * 10^6$	$1.9341 * 10^7$	$2.23319 * 10^7$
τ_c	-590.303	-626.801	-1217.1	$-3.51802 * 10^8$	$-6.44157 * 10^8$	$-9.95959 * 10^8$
α_L	10.2311	27.6715	37.9026	0	$2.58963 * 10^7$	$2.58963 * 10^7$

Panel 2

	β	κ	ψ_H	ψ_{K_1}	$\hat{w}_H^{N*}$	$\hat{r}_1^{N*}$
τ_w	144.591	-0.01494	-0.08256	0	-1	0
τ_k	0	0	0	-0.248132	0	-0.119
τ_c	0	0	0	0	0	0
α_L	2.1658	-0.00014	0	0	0	0

Note: numerical simulation are based on parameter values: $\alpha_1 = 0.119$, $\alpha_L = 38$

migration of unskilled labour κ and the skill factor sought by them, *i.e.*, $\frac{\partial \hat{\kappa}}{\partial \beta} < 0$. Since, in this numerical example, $\frac{\partial \beta^*}{\partial \alpha_L} > 0$, it must be the case that $\frac{\partial \hat{\kappa}}{\partial \beta}\frac{\partial \beta^*}{\partial \alpha_L} < 0$ Thus, the positive effect of an increase in α_L on the socially optimal level of β – *i.e.*, $\frac{\partial \beta^*}{\partial \alpha_L} > 0$ – implies simultaneously a reduction in κ. Secondly, (12) and (14) imply an increase in the relative shadow value/price of human capital λ_{hb} in the formal sector *i.e.*, $\frac{\partial \lambda^*_{hb}}{\partial \alpha_L} > 0$. Since κ is increasing in λ_{hb}, *i.e.*, $\frac{\partial \hat{\kappa}}{\partial \lambda_{hb}} > 0$, hence the increase in the relative shadow price of human capital incentivises more migration from the informal leading to greater human capital formation in the formal sector. Thus, $\frac{\partial \hat{\kappa}}{\partial \lambda_{hb}}\frac{\partial \lambda^*_{hb}}{\partial \alpha_L} > 0$. The net change in socially optimal level of migration κ^* due to an increase in α_L in this numerical example, as seen in Table 3, is positive – the negative influence of increase in the skill factor of the migratory labour force on the socially optimal level of κ is outweighed by the influence that the increase in the shadow price of human capital has on it.

Table 3 shows that the socially optimal value of γ, given by γ^*, is positive, and it increases with increase in the productivity of the unskilled labour force in the informal sector. This implies that $\gamma + n$ is greater than the population rate of growth n in the informal sector. Hence, $\psi_H = \gamma + n$ is also the long-run rate of growth of human capital in the formal sector and it increases with increase in α_L in this numerical example; precisely, it follows from Remarks 1 and 2 and (12) and (14) that

$$\frac{\partial \psi^*_H}{\partial \alpha_L} = \frac{\partial \psi_H}{\partial \tau_w}\frac{\partial \tau^*_w}{\partial \alpha_L} + \frac{\partial \psi_H}{\partial \alpha_L} = \frac{1}{\theta}\left[\frac{\partial \lambda_{hb}}{\partial \tau_w}\frac{\partial \tau^*_w}{\partial \alpha_L} + \frac{\partial \lambda_{hb}}{\partial \alpha_L}\right] = \frac{1}{\theta}\frac{\partial \lambda^*_{hb}}{\partial \alpha_L} > 0$$

In this numerical example, as seen in Table 3, an increase in α_L implies a reduction in socially optimal value of τ_w, the tax on human capital. This in turn increases the shadow value of human capital λ_{hb} and hence the share of investment in human capital, which is also the long-run rate of growth of human capital ψ_H.

In contrast, the long-run rate of growth of physical capital in the formal sector, ψ_{k_1}, is unaffected by increases in α_L. This is because, recalling its definition in (3) and (2) and (14), the shadow value of physical capital $\lambda_{k_1 b}$ is unaffected by τ_w and α_L. Hence, the physical capital investment per unit stock of physical capital in the formal sector is not affected by changes in the productivity of the unskilled labour in the informal sector.

The effect of increase in productivity of the unskilled labour on the optimal net return to human capital is obtained from Remarks 1 and 2 and is positive in this numerical example as seen in (Table 3):

$$\frac{\partial \hat{w}^{N^*}_H}{\partial \alpha_L} = \frac{\partial \hat{w}^N_H}{\partial \tau_w}\frac{\partial \tau^*_w}{\partial \alpha_L} + \frac{\partial \hat{w}^N_H}{\partial \alpha_L} = \frac{\partial \hat{w}^N_H}{\partial \tau_w}\frac{\partial \tau^*_w}{\partial \alpha_L} > 0$$

It follows from the definition of the net (after tax) returns to human capital that $\hat{w}_H^N$ will fall as τ_w increases – see also Remark 1. Hence, $\frac{\partial \hat{w}_H^N}{\partial \tau_w} < 0$. Since Table 3 shows that the socially optimal level of tax on human capital decreases with increase in α_L, the socially optimal returns to human capital increases with increase in α_L. However, since increases in α_L have no impact on socially optimal level of taxation of formal sector's physical capital ($\frac{\partial \tau_k}{\partial \alpha_L} = 0$ as seen in Table 4), the net return to physical capital in the formal sector $\hat{r}_1^N$ remains unchanged as α_L increases: $\frac{\partial \hat{r}_1^{N^*}}{\partial \alpha_L} = 0$.

6.1.2 The net social marginal benefits of taxing human capital at the optimum and its decomposition.

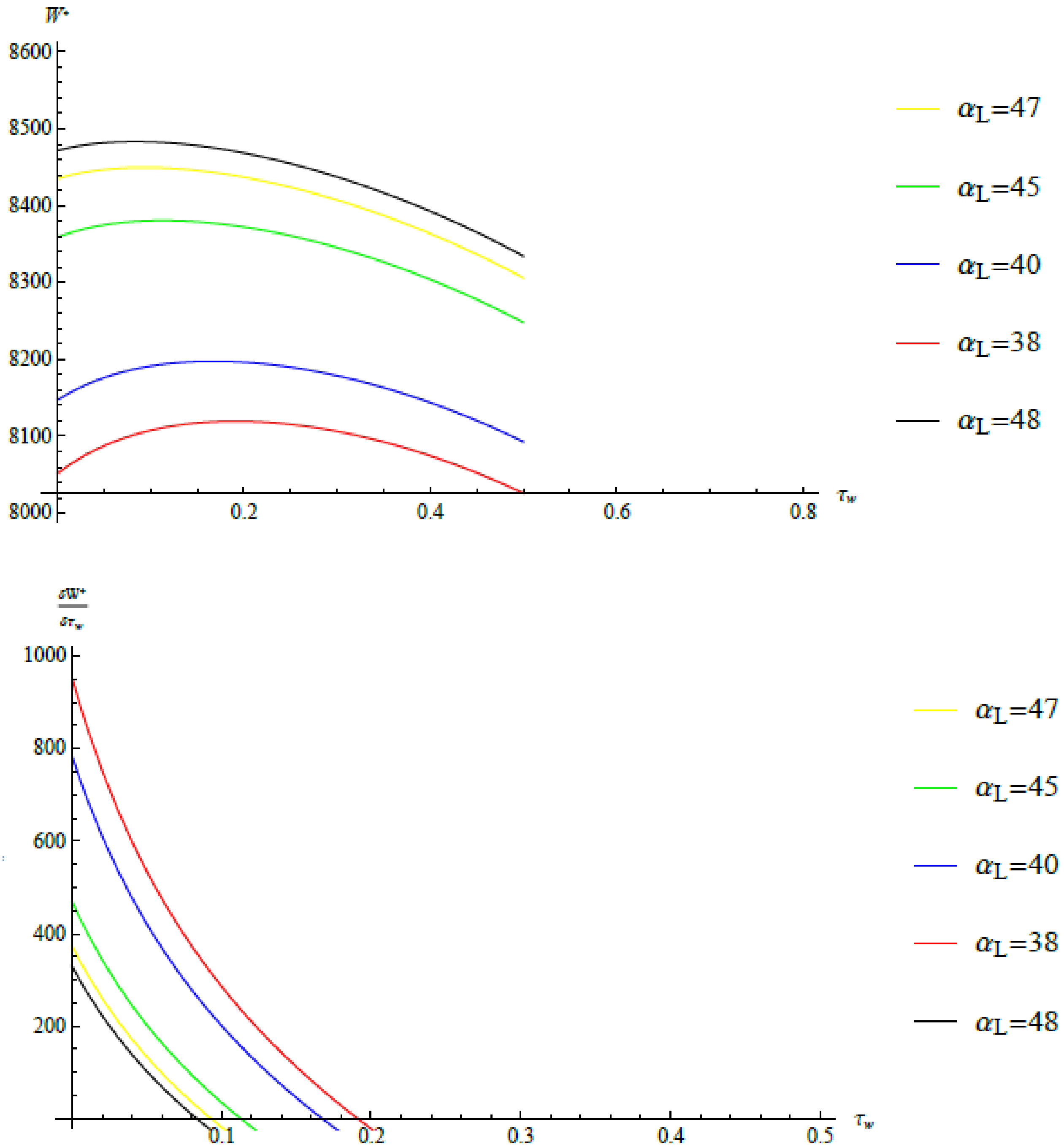
The results presented in Table 3 show that τ_w^* is positive for each of the five values of α_L, indicating interior solutions to the social welfare maximisation problem (18) for these different values of α_L. Moreover, the table also shows that $\hat{\tau}_w$ defined in Section 3.2 is negative and the initial transfers T_0^* are all positive implying that constraints (i) and (iii) of the social welfare maximisation problem (18) are non-binding.

Hence, we are in Case 1 of Remark 3, and at the social optima corresponding to the five values of α_L, the first order conditions with respect to τ_w are equalities of the form (22). Such a first order condition with respect to τ_w says that the net social marginal benefit of taxing human capital is zero at the social optimum. In particular, Table 5 reveals that, at this optimum, there is a marginal inter-temporal benefit to the informal sector household from taxing human capital given by $\frac{du_1}{\partial \tau_w} > 0$ that is equal to the marginal inter-temporal cost from taxing human capital to the formal sector given by $-\frac{du_2}{\partial \tau_w} > 0$, so that the net social marginal benefit from taxation of human capital at the social optimum is zero.

Figure 1 plots variations in social welfare and the net social marginal benefit $\frac{\partial \mathbb{W}^*}{\partial \tau_w}$ with respect to τ_w for various values of α_L holding values of τ_k and τ_c at the socially optimal levels, respectively.[24] As seen in Figure 1, for each value of α_L, the social welfare is (strictly) concave and attains a maximum (at least a local one) with respect to τ_w. Moreover, it shifts up as α_L increases. As expected, the figure hence shows that, for each value of α_L, the net social marginal benefit curve is negatively sloped and intersects the X axis at the level of τ_w where the social welfare function attains its maximum. Moreover, there is a leftward shift in the net social marginal benefits curve as α_L increases, indicating that the optimal tax on human capital will fall as α_L increases.

[24] For each value of α_L, Table 3 shows that the socially optimal levels of τ_k and τ_c are zero.

Figure 1: The graphs of social welfare and net social marginal benefit with respect to human capital taxation, at varying productivity of informal sector labour force



Panel 1 of Table 4 provides the derivatives of welfares of the formal and informal sector households denoted by u_1 and u_2; social welfare $\mathcal{W} = u_1 + u_2$; $\mathcal{C}_{110}$; $\mathcal{C}_{120}$; and $\mathcal{C}_{10} = \mathcal{C}_{110} + \mathcal{C}_{120}$ with respect to relevant economic variables listed in the first column of the table.[25]

Panels 1 and 2 of Table 4 have been employed to construct Table 5. This table provides details of the components of the net social marginal benefits (NSMB) at the optimum and the decomposition of the factors causing the leftward shift in this curve, evaluated at $\alpha_L = 38$, when α_L increases.[26] The table indicates that, evaluated at the social optimum, there are social marginal benefits from increasing τ_w because of the indirect influence it has on migration and the choice of skill factor for the migrating labour force. However, there are also social marginal costs associated with an increase in τ_w primarily due to its impacts on long-run growth rate and the net return to human capital.

For example, Table 4 reveals that, at the social optimum, increase in κ, which we interpret as reflecting the extent of migration of unskilled labour, has a detrimental effect on social welfare ($\frac{\partial \mathcal{W}}{\partial \kappa} < 0$). As indicated by this table, only initial level of private consumption $\mathcal{C}_{120}$ of the informal sector household is adversely affected by migration, which in turn, by affecting $\mathcal{C}_{10}$, also affects the equilibrium supply and consumption of the public good adversely. As a result, inter-temporal welfares of both types of households are adversely affected. In particular, welfare of the informal sector household is affected possibly due to the high opportunity cost in terms of the output lost in the informal sector due to migration and the increased cost of education as more people migrate for a fixed value of β. In this scenario, an increase in τ_w is welfare improving as it has a negative impact on migration at the optimum (Table 4 shows that $\frac{\partial \kappa}{\partial \tau_w} < 0$). Thus, Table 5 shows that there is a positive marginal social benefit from increasing τ_w at the optimum due to its negative effect on socially detrimental migration in this case (*i.e.*, $\frac{\partial \mathcal{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_w} > 0$).

Similarly too, Table 4 and Table 5 can be used to explain the social marginal benefit from increasing τ_w because of the positive influence it has on β and the positive effect β has on social welfare at the optimum.[27]

Table 4 reveals that, while an increase in ψ_H, the investment per unit human capital stock, has a positive impact on the inter-temporal welfare of the formal sector household, which owns a disproportionate amount of the initial endowment of the human capital, it has a negative impact

[25] DM show that the inter-temporal welfare of each household u_m is a function of the initial value of its private consumption, which has been shown to be an increasing function of $\mathcal{C}_{1m0}$, and a public good whose level is proportional to $\mathcal{C}_{10}$.

[26] A similar decomposition could also be done, when evaluating the same at other values of α_L.

[27] See Section 4.3 for a description of the impact of β and other economic variables on social welfare.

Table 5: Decomposition of net social marginal benefits, NSMB

$\alpha_L = 38$, $\alpha_1 = 0.119$	Decomposition	Derivatives with respect to α_L		
$\frac{\partial W}{\partial \beta}\frac{\partial \beta}{\partial \tau_w}$	25.88	-1.11	$\frac{du_1}{d\tau_w}$	-625.656
$\frac{\partial W}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_w}$	158.84	-7.43	$\frac{du_2}{d\tau_w}$	625.656
$\frac{\partial W}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_w}$	-97.90	-8.32	$\frac{d^2u_1}{d\tau_w d\alpha_L}$	-5.37412
$\frac{\partial W}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_w}$	0.00	0.00	$\frac{d^2u_2}{d\tau_w d\alpha_L}$	-24.7892
$\frac{\partial W}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_w}$	-3406.96	79.51		
$\frac{\partial W}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_w}$	0.00	0.00		
$\frac{\partial W}{\partial \tau_w}$	3320.15	-92.81		
NSMB of τ_w (sum)	0.00	-30.16		

$\alpha_L = 38$	$\alpha_1 = 0.119$ Decomposition	$\alpha_1 = 0.2$ Decomposition
$\frac{\partial W}{\partial \beta}\frac{\partial \beta}{\partial \tau_k}$	0.00	0.00
$\frac{\partial W}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_k}$	0.00	0.00
$\frac{\partial W}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_k}$	0.00	0.00
$\frac{\partial W}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_k}$	-1.41	-1.803675698
$\frac{\partial W}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_k}$	0.00	0.00
$\frac{\partial W}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_k}$	-35.90	-55.2946
$\frac{\partial W}{\partial \tau_k}$	34.09	52.0901
NSMB of τ_k (sum)	-3.21	-5.01

$\alpha_L = 38$, $\alpha_1 = 0.119$	Decomposition		
$\frac{\partial W}{\partial \beta}\frac{\partial \beta}{\partial \tau_c}$	0.00	$\frac{\partial W}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_c}$	0.00
$\frac{\partial W}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_c}$	0.00	$\frac{\partial W}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_c}$	0.00
$\frac{\partial W}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_c}$	0.00	$\frac{\partial W}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_c}$	0.00
$\frac{\partial W}{\partial \tau_c}$	-1217.10		
NSMB of τ_c (sum)	-1217.10		
$\frac{\partial U_1^*}{\partial \tau_c}$	-590.303	$\frac{\partial U_2^*}{\partial \tau_c}$	-626.801

on the inter-temporal welfare of the informal sector household. With limited income, investment in capital formation has to come at the cost of reduction in consumption expenditures. The welfare cost of the latter must offset the welfare gains from the former for the informal sector household.

In the net, Table 4 reveals that an increase in ψ_H has a positive impact on social welfare at the optimum ($\frac{\partial \mathcal{W}}{\partial \psi_H} > 0$). But since the table also shows that the increase in τ_w has a decreasing effect on ψ_H at the optimum ($\frac{\partial \psi_H}{\partial \tau_w} < 0$), there is a marginal social cost that an increase in τ_w imposes due to its impact on ψ_H ($\frac{\partial \mathcal{W}}{\partial \psi_H}\frac{\partial \psi_H}{\partial \tau_w} < 0$). See Table 5.

Similarly too, as revealed by Tables 4 and 5, we can decompose and understand the social marginal cost of an increase in τ_w at the optimum due to its impact on the net return to human capital $\hat{w}_H^N$. Precisely, an increase in τ_w reduces the net return to human capital: $\frac{\partial \hat{w}_H^N}{\partial \tau_w} < 0$, while, as seen in Table 4, an increase in $\hat{w}_H^N$ has a positive impact on social welfare. Hence, $\frac{\partial \mathcal{W}}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_w} < 0$.

Table 4 also reveals that the direct (residual) impact of increasing τ_w on social welfare at the optimum is positive, *i.e.*, $\frac{\partial \mathcal{W}}{\partial \tau_w} > 0$. While both households gain from the positive impact that a rise in τ_w has in reducing the cost (including the adjustment cost) of investing in human capital by reducing the relative shadow price of human capital, [28] the informal sector household has an additional direct benefit from an increase in τ_w, namely, the increase in redistributive transfer that, ceteris-paribus, such an increase in taxation makes possible by increasing the government's tax revenue. This is evident from Table 4 that shows that the direct effect of increase in τ_w on the inter-temporal welfare of both households is positive, with the impact being greater for the formal sector household.

Finally, Table 5 shows that, at the social optimum, the net social marginal benefit of increasing τ_w (NSMB of τ_w) obtained from summing up all the impacts – direct and indirect – of increase in τ_w on social welfare is zero.

6.1.3 The comparative statics of increases in α_L – the impact on optimal taxation of human capital.

Table 5 shows that, as α_L increases, the marginal social benefit from taxation of human capital accruing to the informal sector household decreases and the marginal social cost from taxation of human capital of the formal sector household increases, *i.e.*, $\frac{d^2 u_m}{d\tau_w d\alpha_L} < 0$ for both $m = 1, 2$.

[28] Recall, for example, that the cost of investment in human capital is $I_{hm}\left(1+\frac{\theta I_{hm}}{2H}\right) = H_m \frac{I_{hm}}{H_m}\left(1+\frac{\theta I_{hm}}{2H_m}\right) = H_m\left(\frac{\lambda_{hb}^2-1}{\theta}\right) = \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)$.

Thus, the net social marginal benefit from taxation of human capital falls as α_L increases. This explains the leftward shift in the net social marginal benefit curve with respect to τ_w when α_L increases that is seen in Figure 1.

To study the factors that lead to such a shift in the net social marginal benefit curve with respect to τ_w, we evaluate the derivatives with respect to α_L of the various components of the net social marginal benefit of τ_w at the optimum corresponding to $\alpha_L = 38$.

Table 5 shows that an increase in α_L, starting from the social optimum corresponding to $\alpha_L = 38$, reduces the social marginal benefit from increasing τ_w due to its impacts on migration and choice of skill factor *i.e.*, $\frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_w}\right) < 0$ and $\frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_w}\right) < 0$. Similarly too, the direct effect of increase in τ_w on social welfare is lower when α_L is increased, *i.e,* $\frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \tau_w}\right) < 0$. Furthermore, the table also shows that an increase in α_L also increases the social marginal cost of increasing τ_w due to the impact the increase in τ_w has on social welfare by impacting the long-run rate of growth of human capital: $\frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \hat{\psi}_H}\frac{\partial \psi_H}{\partial \tau_w}\right) < 0$. Together these channels contribute to a decline in the net marginal social benefits with respect to τ_w at the optimum when α_L increases – *i.e.*, they have a tendency to shift the net social marginal benefit curve with respect to τ_w to the left.

On the other hand, an increase in α_L starting from the social optimum corresponding to $\alpha_L = 38$ has a decreasing effect on the social marginal cost of τ_w due to its impact on the net return to human capital: $\frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_w}\right) > 0$. This has a tendency to increase the net social marginal benefit of τ_w at the optimum when α_L increases, *i.e.*, it has a tendency to shift the net social marginal benefit curve with respect to τ_w to the right.

Table 5 shows that the influence of factors causing a decline in net social marginal benefit of τ_w at the optimum due to an increase in α_L offset the influence of the factor causing an increase in social marginal benefit of τ_w, leading to an overall leftward shift in the net social marginal benefit curve of τ_w:

$$\sum_{m=1}^{2}\frac{d^2 u_m}{d\tau_w d\alpha_L} = \frac{\partial}{\partial \alpha_L}\left(\frac{\partial \mathbb{W}}{\partial \kappa}\frac{\partial \kappa}{\partial \tau_w} + \frac{\partial \mathbb{W}}{\partial \beta}\frac{\partial \beta}{\partial \tau_w} + \frac{\partial \mathbb{W}}{\partial \hat{\psi}_H}\frac{\partial \psi_H}{\partial \tau_w} + \frac{\partial \mathbb{W}}{\partial \tau_w} + \frac{\partial \mathbb{W}}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^N}{\partial \tau_w}\right) = -30.16 < 0.$$

As seen in Figure 1, this implies that the optimal value of τ_w falls as α_L increases in our numerical example.

6.1.4 Changes in the financing of the redistributive transfer with increases in α_L.

As seen in Table 3, the initial income of the formal sector household, y_1^*, is higher than the informal sector household, y_2^*, for $\alpha_L = 38$. As the productivity of labour in the informal sector, α_L, increases, the base income of this household also increases. Nevertheless, Table 3 demonstrates that the base income differential between the formal and the informal sector households does not drop significantly as α_L increases. It decreases from 719068000 to 642668000 as α_L increases from 38 to 48.[29] So, one expects that the socially optimal extent of redistribution too should not decline significantly when α_L increases in this numerical example.

That there is redistribution from the initially richer formal sector to the initially poorer informal sector under the socially optimal fiscal policy, is evident from the fact that, for each value of α_L, at the tax equilibrium that maximises social welfare, the welfare of the informal sector household is higher than that of the formal sector household. The welfare differential after redistribution does not change much as α_L increases. It ranges between 245 and and 274 utils.

This redistribution is achieved through the transfer instrument T. Focusing on its value at the initial time point, it is given from (11) as follows:

$$T(0) \quad = \quad \tau_b r_b B(0) + \tau_k r_1 K_1(0) + \tau_w w_1 H(0) + \tau_c C_1(0) + G_1(\mathcal{I}A_1 - 1) - G_c(0) - p\,(\mathcal{P})\,G_2$$

As seen in Table 3 the importance of this instrument does not diminish with increase in the productivity of the informal sector labour force α_L; rather, $T(0)$ increases as α_L increases. Table 3 also reveals a striking feature about the financing of the transfer. It changes as α_L increases. To see this, note that, in this model, in addition to the traditional instruments of taxation such as capital and consumption taxation, government expenditure on infrastructure in the formal sector G_1 is also a source of governmental revenue. This is because G_1 yields profit/economic rent to the formal sector that is assumed to be fully taxed away by the government in this numerical example. The revenue earned through this is $\mathcal{I}A_1G_1$, the government expenditure on infrastructure in the formal sector, which is a multiple of G_1.

Table 3 reveals that as α_L increases, social welfare maximisation implies that the reliance on capital taxation as a source of government revenue falls; precisely, the tax rate on human capital, τ_w^*, declines from 19% to 8%. Instead, G_1 plays an increasing role in generating revenue for the government due to the taxation of the economic rent that it generates in the formal

[29] Recall that the formal sector good is the numeraire, so income is measured in units of good 1.

sector. We see that G_1^* increases from 135950000 to 206205000 as α_L increases from 38 to 48.[30] Thus, the transfer to the informal sector household is increasingly financed from taxation of the economic rent generated in the formal sector.

Thus, in institutions where alternative instruments for generation of tax revenue to meet redistributive objectives exist, optimal capital taxation policies aim more and more to promote the growth objectives. Indeed, as seen in Table 3, τ_w^* falls and the optimal long-run growth rate of human capital ψ_H^* increases from 4.2% to 5.1% as α_L increases from 38 to 48. Thus, when productivity differentials between the formal and informal sector decrease, the brunt of financing redistribution begins to bear more heavily on the other tax instruments such as those that tax economic rent generated in the formal sector rather than capital taxation.

6.1.5 The impact of changes in α_L on the socially optimal taxation of formal-sector physical capital and consumption.

Table 3 shows that τ_k^* and τ_c^* are zero for all the five values of α_L considered, indicating corner solutions for τ_k and τ_c in the social welfare maximisation problem (18). Moreover, since $\hat{\tau}_k$ is negative and the transfers are all positive, constraints (ii) and (iii) of problem (18) are non-binding. Hence, we are in Case 2 of Remark 3, and at the social optima corresponding to the five values of α_L, the first order conditions with respect to τ_k and τ_c are inequalities of the form (23). Such a first order condition says that the net social marginal benefits of taxing physical capital or consumption in the formal sector are negative at the social optimum. Figures 2 and 3 plot the graph of social welfare with respect to τ_k and τ_c, respectively, for various values of α_L, the productivity of the informal sector labour force, holding values of τ_w fixed at the corresponding socially optimal levels, respectively. They show that the graph of the welfare function shifts up as α_L increases – reflecting positive income effects for higher values of α_L; however, the graphs are always negatively sloped with the social welfare achieving its maximum at $\tau_k = \tau_c = 0$ for all the five values of α_L considered. Table 5, which assumes $\alpha_L = 38$, shows that, at the social optimum, the net marginal social benefits from taxation of physical capital (NMSB of τ_k) in the formal sector is negative because increasing τ_k at the social optimum adversely impacts net returns to physical capital, $\hat{r}_1^N$, and its the long-run growth, ψ_{k_1}, both of which increase social welfare.[31] When $\alpha_1 = 0.119$, these marginal social costs are -35.90 and -1.41, respectively, and offset the total marginal benefit in the form of increases in government's

[30] G_1 is measured also in units of good 1.

[31] $\frac{\partial \hat{r}_1^N}{\partial \tau_k} < 0$ and $\frac{\partial \psi_{k_1}}{\partial \tau_k} < 0$, while $\frac{\partial \mathcal{W}^*}{\partial \hat{r}_1^N} > 0$ and $\frac{\partial \mathcal{W}^*}{\partial \psi_{k_1}} > 0$. Hence, $\frac{\partial \mathcal{W}^*}{\partial \hat{r}_1^N}\frac{\partial \hat{r}_1^N}{\partial \tau_k} < 0$ and $\frac{\partial \mathcal{W}^*}{\partial \psi_{k_1}}\frac{\partial \psi_{k_1}}{\partial \tau_k} < 0$.

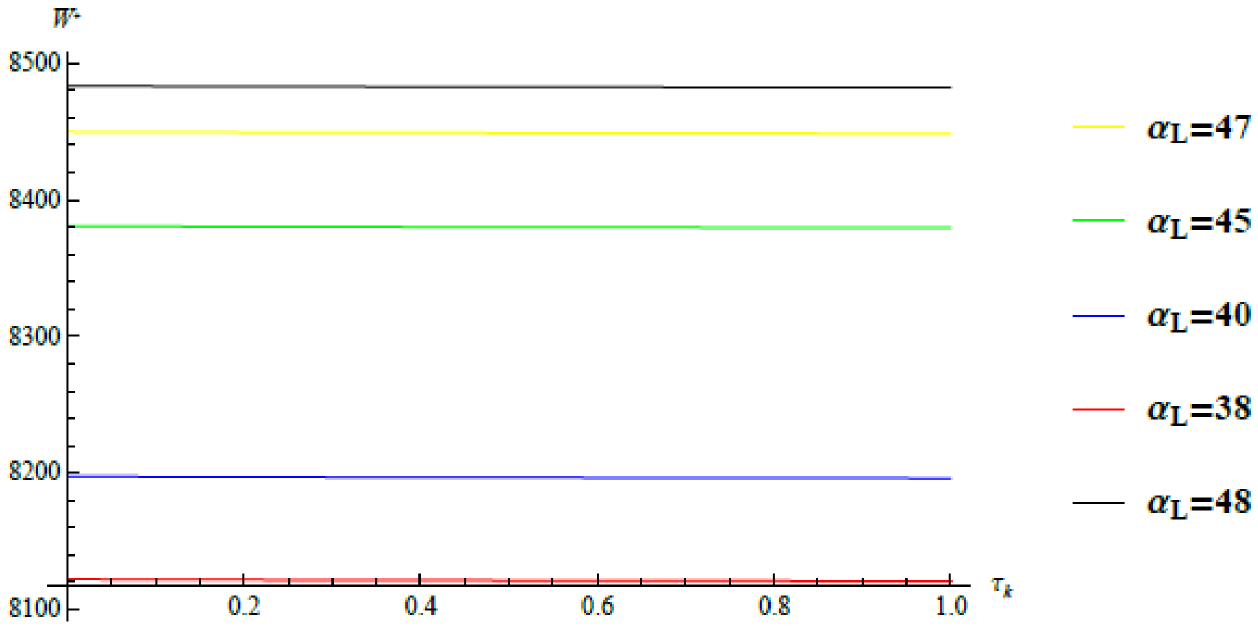


Figure 2: The graph of social welfare with respect to physical capital taxation, at varying productivity of informal sector labour force

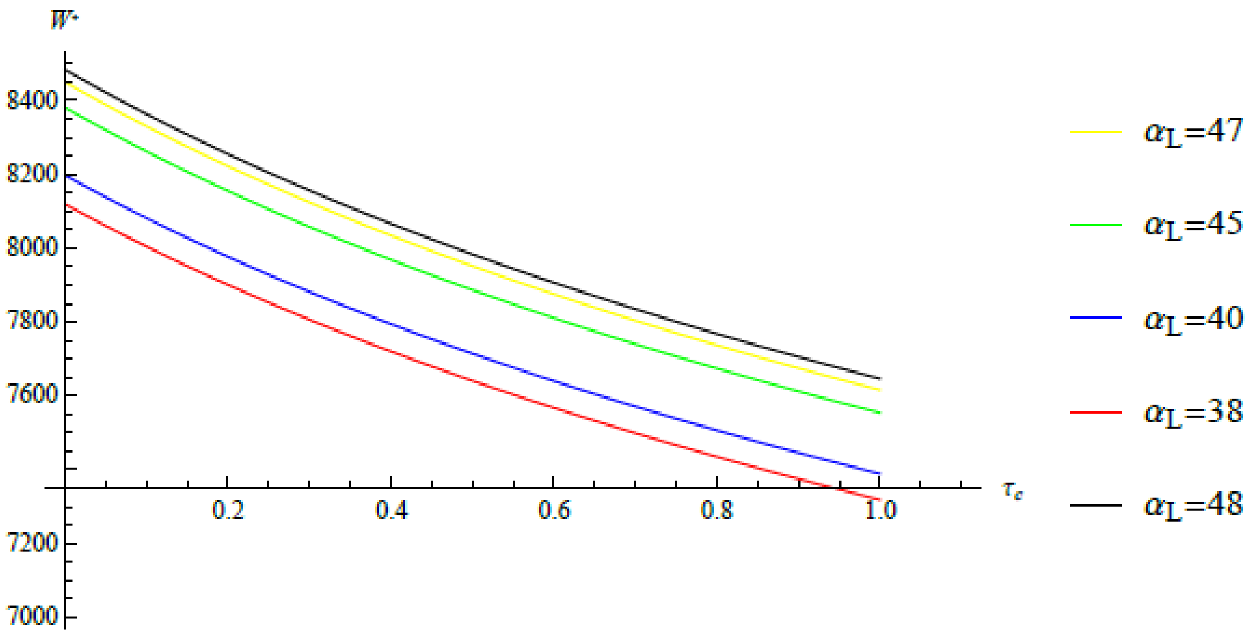


Figure 3: The graph of social welfare with respect to commodity taxation, at varying productivity of informal sector labour force

tax revenue and reductions in the adjustment costs of investment from taxing physical capital in the formal sector, which is 34.09. Thus the net social marginal benefit from taxation of physical capital in the formal sector is -3.21, which is negative.

Note that changes in τ_c have no impact on migration, skill factor, growth rates of and net returns to physical and human capital. The consumption tax is primarily a revenue generating instrument. To the extent it taxes consumption in both the formal and the informal sector, increases in τ_c have an adverse impact on social welfare by adversely affecting the welfare of households of both sectors. Table 5 shows this in the case of $\alpha_L = 38$ – the derivatives of intertemporal welfare of the formal and informal sector households with respect to the consumption tax, evaluated at the social optimum, are given by $\frac{\partial u_1}{\partial \tau_c} = -590.303$ and $\frac{\partial u_2}{\partial \tau_c} = -626.801$, respectively. Hence, Table 5 shows that, in our numerical example, the net social marginal benefit of taxing consumption (NMSB of τ_c) is negative, precisely, it is $\frac{\partial u_2}{\partial \tau_c} + \frac{\partial u_2}{\partial \tau_c} = -1217.10$. This implies that the marginal social cost of τ_c in the form of reduction in consumption induced by it is more than offset by its marginal social benefit in the form of increases in the government's tax revenue.

6.2 Comparative statics with respect to changes in the productivity of physical capital in the formal sector, α_1.

Table 6 presents all the comparative static results with respect to variations in α_1, the productivity of the physical capital in the formal sector. Here, we are assuming that the value of α_L is 38.[32] We allow α_1 to take six values, namely, 0.119, 0.15, 0.18, 0.20, 0.25, and 0.28.

6.2.1 The separability between optimal taxation of human and physical capital in the formal sector.

It is to be noted that the indirect and direct marginal social benefits from taxing human capital, given by $\frac{\partial W^*}{\partial \beta}\frac{\partial \beta^*}{\partial \tau_w}$, $\frac{\partial W^*}{\partial \kappa}\frac{\partial \kappa^*}{\partial \tau_w}$, $\frac{\partial W^*}{\partial \psi_H}\frac{\partial \psi_H^*}{\partial \tau_w}$, $\frac{\partial W^*}{\partial \hat{w}_H^N}\frac{\partial \hat{w}_H^{N^*}}{\partial \tau_w}$, and $\frac{\partial W^*}{\partial \tau_w}$, are all independent of α_1. Hence Table 6 indicates that the optimal level of tax on human capital τ_w^* and the socially optimal levels of κ, β, $\hat{w}_H^N$ and ψ_H are invariant to changes in α_1. These continue to be fixed at the values they took in Table 3 when α_L was equal to 38.

[32] A similar analysis can be done for any other value of α_L also.

Table 6: Impact of changes in productivity of formal sector physical capital on optimal fiscal policy and other economic variables

	Values of α_1					
	0.119	**0.15**	**0.18**	**0.2**	**0.25**	**0.28**
τ_w^*	0.190925	0.190925	0.190925	0.190925	0.190925	0.190925
τ_k^*	0	0.243331	0.394564	0.46504	0.585473	0.634873
τ_c^*	0	0	0	0	0	0
T_0^*	1052200	0	0	0	0	0
G_1^*	135950000	135428000	135044000	134800000	134207000	133858000
G_2^*	1849890000	1849890000	1849890000	1849890000	1849890000	1849890000
κ^*	0.00293312	0.00293312	0.00293312	0.00293312	0.00293312	0.00293312
β^*	88.118	88.118	88.118	88.118	88.118	88.118
γ^*	0.02655	0.02655	0.02655	0.02655	0.02655	0.02655
$\hat{w}_H^{N*}$	0.809075	0.809075	0.809075	0.809075	0.809075	0.809075
$\hat{r}_1^{N*}$	0.119	0.11350035	0.10897848	0.106992	0.10363175	0.10223556
$\psi_{k_1}^*$	0.042041685	0.031691	0.0243929	0.0214319	0.0166947	0.0148145
ψ_H^*	0.04155	0.04155	0.04155	0.04155	0.04155	0.04155
$\mathcal{C}_{11}(0)$	351865000	350871877	351379000	351298000	351168000	351116000
$\mathcal{C}_{12}(0)$	644834000	646266101	645319000	645401000	645531000	645583000
$\mathcal{C}_1(0)$	996699000	997137977	996698000	996699000	996699000	996699000
U_1^*	3937.77	3937.45	3937.23	3937.13	3936.99	3936.93
U_2^*	4183.67	4183.86	4183.99	4184.04	4184.12	4184.16
$U_2^* - U_1^*$	245.9	246.41	246.76	246.91	247.13	247.23
$U^* = W^*$	8121.44	8121.31	8121.22	8121.17	8121.11	8121.09
y_1^*	1010690000.00	1013480000	1016180000	1017980000	1022480000	1025180000
y_2^*	291622000.00	291932000	292232000	292432000	292932000	293232000
$y_1^* - y_2^*$	719068000	721548000	723948000	725548000	729548000	731948000
$\psi_{\mathcal{C}}^*$	0.028888889	0.02888889	0.02888889	0.028888889	0.028888889	0.028888889
$\psi_{k_2}^*$	0.028888889	0.02888889	0.02888889	0.028888889	0.028888889	0.028888889
ψ_B	0.042041685	0.04155	0.04155	0.04155	0.04155	0.04155
ψ_{Y_1}	0.042041685	0.04155	0.04155	0.04155	0.04155	0.04155
ψ_{Y_2}	0.028888889	0.02888889	0.02888889	0.028888889	0.028888889	0.028888889
$\hat{X}^*$	0.002395051	0.00243068	0.00243068	0.00243068	0.00243068	0.00243068
λ_{hb}^*	11.3875	11.3875	11.3875	11.3875	11.3875	11.3875
$\lambda_{k_1}^*$	1.42042	1.31691	1.24393	1.21432	1.16695	1.14814
p^*	0.764006791	0.76400679	0.76400679	0.764006791	0.764006791	0.764006791
$\hat{\tau}_w$	-0.1025	-0.1025	-0.1025	-0.1025	-0.1025	-0.1025
$\hat{\tau}_k$	-0.096638655	0.13	0.275	0.3475	0.478	0.533929
$\underline{\tau}_w$	0	0	0	0	0	0
$\underline{\tau}_k$	0	0.13	0.275	0.3475	0.478	0.533929
τ_k^T	does not exist	0.243331	0.394564	0.46504	0.585473	0.634873

Note: numerical simulation are based on parameter values: $\alpha_L = 38$ and $\alpha_2 = 0.1178$

6.2.2 Characterisation of optimal taxation of formal-sector physical capital and the transfer to the informal sector.

As pointed out in Section 3.2, a necessary condition for tax equilibria to exist for a given configuration of parameters $\langle \alpha_L, \alpha_1, \mathcal{P} \rangle$ is that

$$\tau_k \geq \underline{\tau}_k (\alpha_L, \alpha_1, \mathcal{P}) := \max\{\hat{\tau}_k (\alpha_L, \alpha_1, \mathcal{P}), 0\}, \tag{25}$$

where $\hat{\tau}_k$ is the level of τ_k that solves (9) as an equality. As explained in Section 3.2, given a value of α_1, $\hat{\tau}_k$ is the minimum value that τ_k can take to ensure that the relative shadow price of formal-sector physical capital is real and non-negative. In general, the definition of $\hat{\tau}_k$ permits it to take negative values.[33] Hence, to restrict our actual analysis to the case where tax rates can take only non-negative values and $\lambda_{k_1 b}$ is well defined, we define $\underline{\tau}_k$ as the maximum of zero and $\hat{\tau}_k$. Any tax rate τ_k that is greater than or equal to $\underline{\tau}_k$ will be non-negative and will result in $\lambda_{k_1 b}$ taking non-negative real values.

In our model, the transfer payment to the informal sector is also required to be non-negative. To identify the range for τ_k in which both (i) the transfer will be non-negative and (ii) the relative shadow price of formal-sector physical capital, $\lambda_{k_1 b}$, will take real and non-negative values, we define τ_k^T as the solution of the equation

$$\hat{T}_0 \left(\tau_w, \tau_k^T, \tau_c, \alpha_L, \alpha_1, \mathcal{P}\right) = 0 \quad \text{for} \quad \tau_k^T \in [\underline{\tau}_k, 1]$$

for fixed values of τ_w, τ_c, α_L, α_1, and $\mathcal{P}$. This means that τ_k^T is a function of τ_w, τ_c, α_L, α_1, and $\mathcal{P}$: $\tau_k^T (\tau_w, \tau_c, \alpha_L, \alpha_1, \mathcal{P})$. Note that τ_k^T may not exist – there may be cases where there is no tax rate $\tau_k \in [\underline{\tau}_k, 1]$ such that the transfer is zero.

Table 6 provides the values of $\hat{\tau}_k$, $\underline{\tau}_k$, and τ_k^T for the six different values of α_1 taken. Figure 4 plots the initial transfer function $\hat{T}_0 (\tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P})$ for each of the six values of α_1. It is clear that τ_k^T is that non-negative tax rate on physical capital τ_k where the graph of $\hat{T}_0 (\tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P})$ cuts the horizontal axis. Figure 4 shows that the initial transfer function $\hat{T}_0$ is increasing in the interval $[\underline{\tau}_k, 1]$ for each of the six values of α_1. When τ_k^T exists, this implies that the transfer is non-negative for only those values of τ_k that are at least as big as τ_k^T:

$$\hat{T}_0 (\tau_w, \tau_k, \tau_c, \alpha_L, \alpha_1, \mathcal{P}) \geq 0 \iff \tau_k \geq \tau_k^T \quad \text{whenever } \tau_k^T \in [\underline{\tau}_k, 1] \text{ exists.} \tag{26}$$

[33]For example, in the case where $\alpha_1 = 0.119$. See Table 3 or Table 6

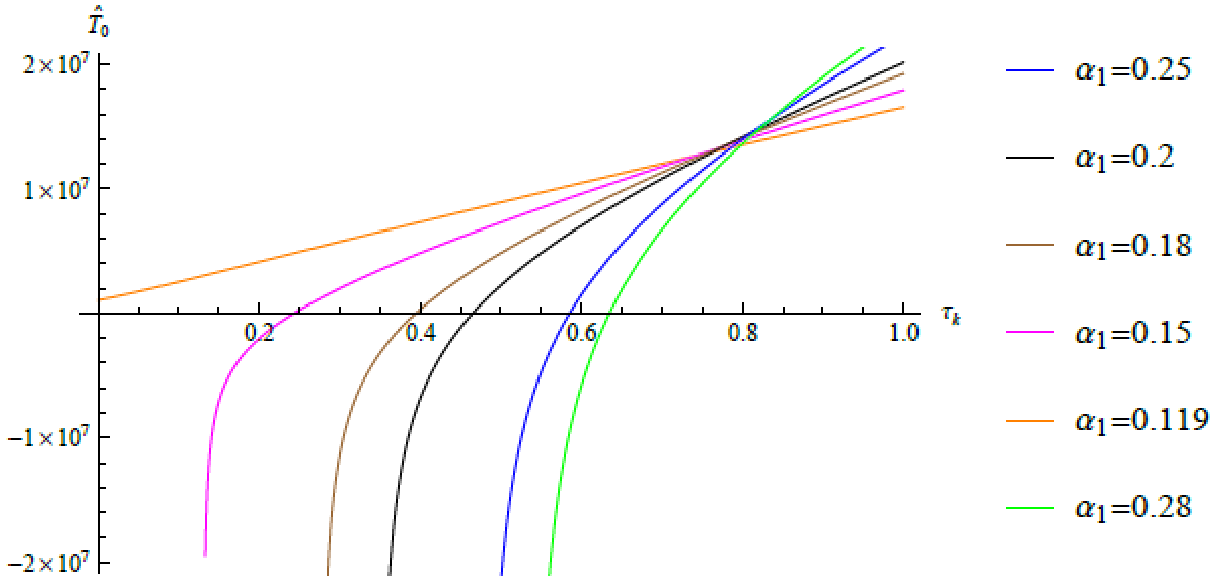


Figure 4: The graph of the initial transfer with respect to physical capital taxation

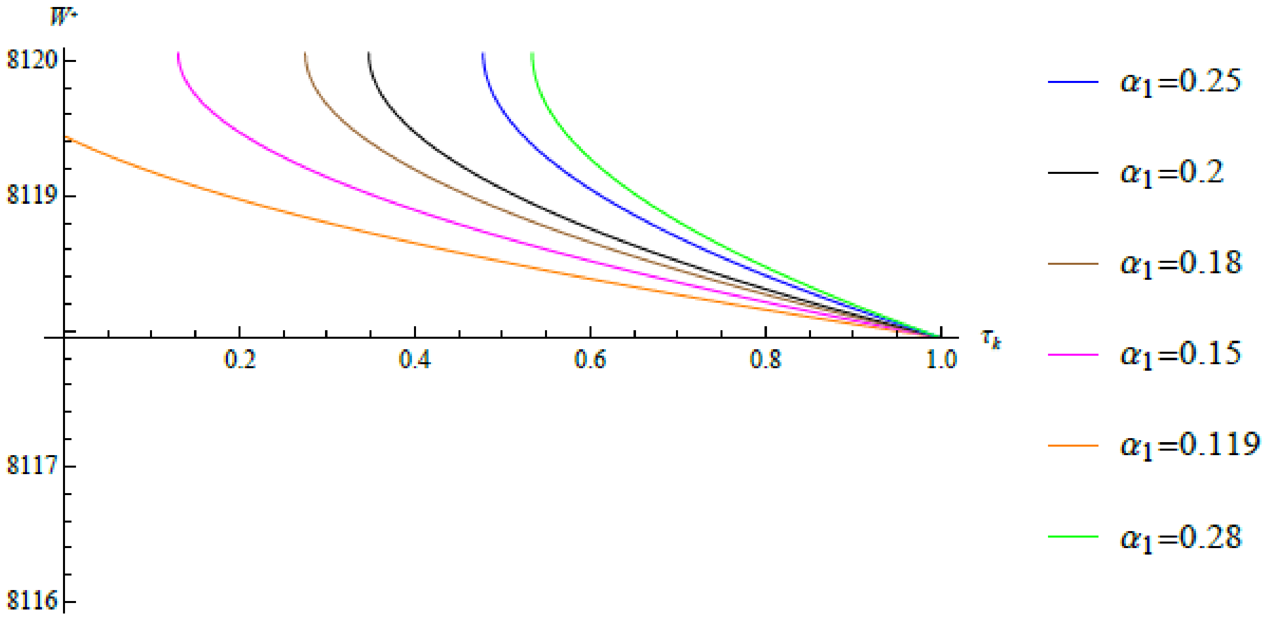


Figure 5: The graph of social welfare with respect to physical capital taxation

Figure 4 shows that τ_k^T does not exist for $\alpha_1 = 0.119$, as the graph of $\hat{T}_0$ for this value of α_1 does not cut the horizontal τ_k axis at a non-negative tax rate.

For all the six values of α_1 in Table 6, Figure 5 shows that social welfare $\mathcal{W}^*$ is decreasing in τ_k for all $\tau_k \geq \underline{\tau}_k$ when τ_w is held fixed at its optimal value $\tau_w^* = 0.190925$ corresponding to $\alpha_L = 38$. Thus, the net social marginal benefit of tax on physical capital in the formal sector is negative. As seen in Table 5, which considers the cases where α_1 takes values 0.119 and 0.20, increases in τ_k indirectly impose social costs by adversely impacting the long-run growth rate and the net return to physical capital in the formal sector. On the other hand, they also directly result in social benefits by positively affecting the government's tax revenue. This table shows that $\frac{\partial \mathcal{W}}{\partial \psi_{k_1}} \frac{\partial \psi_{k_1}}{\partial \tau_k} < 0$, $\frac{\partial \mathcal{W}}{\partial \hat{r}_1^N} \frac{\partial \hat{r}_1^N}{\partial \tau_k} < 0$, and $\frac{\partial \mathcal{W}}{\partial \tau_k} > 0$. The net effect of increasing τ_k is a fall in social welfare. Thus, the net social marginal benefit of τ_k is -3.21 for $\alpha_1 = 0.119$ and -5.01 for $\alpha_1 = 0.2$.

Given the negative net social marginal benefit from taxing physical capital in the formal sector, the optimal tax on physical capital in the formal sector will be the lowest possible value τ_k can take given the value of α_1. Thus, in particular, when when τ_k^T exists, the optimal tax rate will be $\tau_k^* = \tau_k^T$.

The following remark characterises the two types of solutions seen in this numerical example for the optimal tax on physical capital in the formal sector.[34]

Remark 4 *In our numerical example, we find that the social welfare function* $\mathcal{W}^*(\tau_w^*, \tau_k, \tau_c^*, \alpha_L, \alpha_1, \mathcal{P})$ *is decreasing in* τ_k *in the interval* $[\underline{\tau}_k, 1]$ *and the following two cases of optimal taxation on physical capital in the formal sector are seen:*

Case 1. $\tau_k^T \in [\underline{\tau}_k, 1]$ *exists and* $\tau_k^T > \underline{\tau}_k \geq 0$*: In this case, we have*

- *a.* $\tau_k^* = \tau_k^T > 0$, *i.e., we have an interior solution for* τ_k *and*
- *b. Constraint (iii) of the social welfare maximisation problem (18) is binding as definition of* τ_k^T *implies that* $\hat{T}_0(\tau_w^*, \tau_k^*, \tau_c^*, \alpha_L, \alpha_1, \mathcal{P}) = 0$

Case 2. $\hat{\tau}_k < 0$, τ_k^T *does not exist in* $[\underline{\tau}_k, 1]$, *and* $\hat{T}_0(\tau_w^*, \tau_k, \tau_c^*, \alpha_L, \alpha_1, \mathcal{P}) > 0$ *for all* $\tau_k \in [\underline{\tau}_k, 1]$*: In this case, we have*

- *a.* $\tau_k^* = \underline{\tau}_k = 0$, *i.e., we have a corner solution for* τ_k *and*
- *b. Constraint (iii) of the social welfare maximisation problem (18) is non-binding at the optimum, i.e.,* $\hat{T}_0(\tau_w^*, \tau_k^*, \tau_c^*, \alpha_L, \alpha_1, \mathcal{P}) > 0$

[34] This characterization in not exhaustive. In theory, there could also be other possibilities. But these are the only two cases observed in this numerical example.

Cases 1 and 2 above correspond, respectively, to Cases 3 and 2 of Remark 3.

In particular, Figure 5 shows that the solution to social welfare maximisation when $\alpha_1 = 0.119$ corresponds to Case 2 of Remark 4. Here, we have a corner solution for τ_k (*i.e.*, $\tau_k^* = 0$) with the socially optimal value of the initial transfer being strictly positive. As can be seen in Figure 4, the initial transfer function $\hat{T}_0$ is strictly increasing and takes positive values for all $\tau_k \in [0, 1]$. Hence, τ_k^T does not exist in this case.

Figure 5 also shows that the solutions to social welfare maximisation for the remaining five values of α_1 correspond to Case 1 of Remark 4. Here, $\tau_k^T > \underline{\tau}_k$. Since social welfare is decreasing in τ_k, this implies that the optimal tax on physical capital τ_k^* is given by τ_k^T, which is the minimum tax rate for which the shadow price of physical capital in the formal sector is well-defined and the transfer is non-negative. Hence, the optimal value of the initial transfer $\hat{T}_0$ is zero for all these five values of α_1. For tax rates below τ_k^T, either the transfer is negative or $\lambda_{k_1 b}$ is not real.

6.2.3 Impact of changes in α_1 on optimal taxation of physical capital.

Table 6 shows that the optimal tax on formal-sector physical capital increases as the productivity of this input α_1 increases. It increases from 0 to 63% as α_1 increases from 0.119 to 0.28. To understand this, recall that the social welfare is decreasing in τ_k in our numerical example for all the six values of α_1 considered. Hence, the optimal tax on physical capital corresponds to the least feasible value of τ_k given α_1 that ensures a non-negative real value of $\lambda_{k_1 b}$ and a non negative transfer at the initial time point. In our numerical example, as discussed in Remark 4, this least feasible value of τ_k is either given by $\underline{\tau}_k$ or τ_k^T. In this regards, $\underline{\tau}_k = \min\{\hat{\tau}_k, 0\}$ is non-decreasing in α_1. This is because $\hat{\tau}_k$ is increasing in α_1.[35] Figure 5 shows that the graph of social welfare with respect to τ_k, which begins with $\tau_k = \underline{\tau}_k$, shifts to the right as α_1 increases, *i.e.*, $\underline{\tau}_k$ increases as α_1 increases. Similarly, Figure 4 shows that the graph of the initial transfer function $\hat{T}_0$ is increasing in τ_k and shifts down as α_1 increases. Hence, whenever it exists, τ_k^T increases as α_1 increases. This is verified by Table 6.

6.2.4 Mode of redistribution in the presence of optimal taxation of physical capital in the formal sector.

The manner in which redistribution is achieved in the case when $\alpha_1 = 0.119$ has been discussed in Section 6.1. In this case, redistributive nature of the optimal fiscal policy is explicitly seen in

[35] Solving (9) as an equality for $\hat{\tau}_k$, we obtain $\hat{\tau}_k = 1 - \frac{\pi^2 r_b^2 + 2\pi r_b}{2\alpha_1 \pi}$. From this it follows that $\frac{\partial \hat{\tau}_k}{\partial \alpha_1} > 0$.

the form of (i) taxation of the returns from human capital, (ii) 100% taxation of the economic rent generated by government's infrastructural expenditure G_1 in the formal sector, (iiii) and through a positive transfer to the initially more deprived informal sector household.

Table 7: Tax revenues, government expenditures, and transfers in time period zero

α_1	G_2	G_c	Net tax revenue from taxing G_1	Tax revenue from taxing H	Tax revenue from taxing K_1	T(0)
0.119	1413330000	99670	1223550000	190925000	0	1050000
0.15	1413330000	99670	1218850030	190925000	3649970	0
Difference	0	0	-4699970	0	3649970	-1050000

Note: numerical simulation are based on parameter values: $\alpha_L = 38$ and $\alpha_2 = 0.1178$

For the remaining five values of α_1, as seen in Table 6, there is additionally also a tax on returns to formal-sector physical capital, which acts in a direction of further lowering welfare of especially the formal sector household, which holds disproportionately larger amount of the initial endowment of this type of capital. Table 7 shows that while government expenditure on infrastructural activities in the informal sector G_2 and provision of public good G_c do not change when α_1 changes from 0.119 to 0.15[36], the increase in government revenue in the initial period due to taxation of returns from formal-sector physical capital – given by 3649970 – is more than offset by the decrease in the revenue to the government from taxing the profit of the formal sector firms – given by 4699970, both in units of good 1. The net loss in the initial-period government revenue as we move from $\alpha_1 = 0.119$ to $\alpha_1 = 0.15$ is exactly equal to the transfer given to the informal sector household when $\alpha_1 = 0.119$, which is 1050000 in units of good 1. Hence, there is no initial transfer when $\alpha_1 = 0.15$.

Despite not receiving any transfer in time-period zero, as seen in Table 8 the levels of welfare and initial consumption $\mathcal{C}_{12}(0)$ of the informal sector household when $\alpha_1 = 0.15$ are high and remain comparable to those when $\alpha_1 = 0.119$. The absolute differences in initial consumption and welfare for the informal sector household are 285000 and 0.19, respectively. Thus, significant redistribution does happen at the optimum when $\alpha_1 = 0.15$ even when the initial transfer to the informal sector household is zero.

To understand how this redistribution is effected when $\alpha_1 = 0.15$, note from Figure 6 that although the initial transfer to the informal sector household is zero, the transfer increases over time.[37] Note also that the initial consumption $\mathcal{C}_{12}(0)$ in the informal sector household is determined by equation (31) in the definition of a tax equilibrium, which can be interpreted as the present discounted value of it lifetime budget constraint, where the components of this

[36] They remain at 1413330000 and 99669.8 in units of good 1.
[37] Although its level is lower than in the case where $\alpha_1 = 0.119$.

Table 8: Initial welfare and consumption of the informal sector household

α_1	$\mathcal{C}_{12}(0)$	$U_2(0)$
0.119	644834000	4183.67
0.15	645119000	4183.86
Difference	285000	0.19

Note: numerical simulation are based on parameter values: $\alpha_L = 38$ and $\alpha_2 = 0.1178$

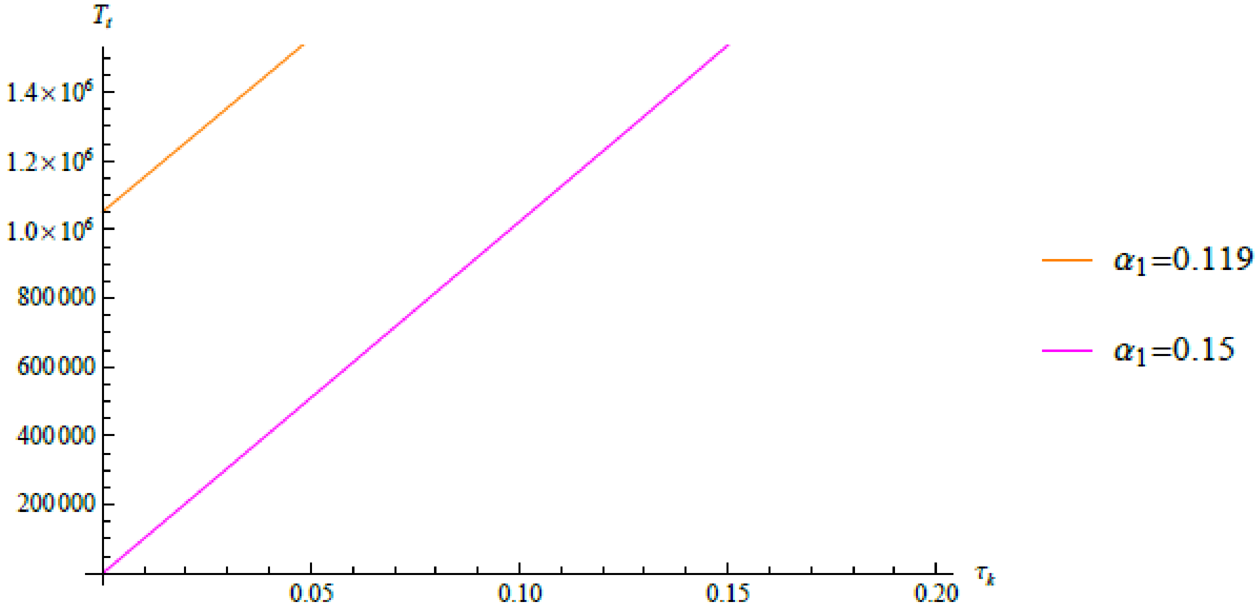


Figure 6:

budget constraint such as net returns from human and physical capital, consumption, grow at different rates. For example, physical capital grows at a rate ψ_{k_1}, while consumption grows at a rate $\psi_{\mathcal{C}}$. Hence, different discount rates will be used for the various components of the budget constraint while computing the present discounted value of the lifetime budget constraint.

Table 9: Present discounted values

α_1	Tax revenue on K_1	Net tax revenue on G_1	Total
0.119	0	13595000000	13595000000
0.15	62597000	13542800000	13605400000
Difference	62597000	-52200000	10400000

Note: numerical simulation are based on parameter values: $\alpha_L = 38$ and $\alpha_2 = 0.1178$

In particular, ψ_{k_1} has to be used as the discount rate to compute the the present discounted value of the government's tax revenue from taxing formal-sector physical capital. Employing this as discount rate, Table 9 shows that the present discounted value of government's tax revenue from physical capital increases from 0 to 62597000 as α_1 increases from 0.119 to 0.15.

At the same time, the present discounted value of revenue from profit taxation (net revenue from G_1) falls from 13595000000 to 13542800000, all in units of good 1. Table 9 shows that, together, the present discounted value of government revenue from G_1 and taxing formal-sector physical capital is 13605400000 when $\alpha_1 = 0.15$, which is very similar to 13595000000, the levels seen when $\alpha_1 = 0.119$. As seen in Table 6, the long-run rate of growth of human capital, ψ_H, and τ_w^* do not change as α_1 changes. This implies that the present discounted value of government's tax revenue from human capital does not change with increase in α_1.

Given no change in the government expenditures on G_2 and G_c and no change in tax revenue from taxing human capital, the above implies that the difference in the present discounted value of transfer when $\alpha_1 = 0.15$ and the same when $\alpha_1 = 0.119$ is exactly equal to the difference in the present discounted values of government revenue from G_1 and taxing formal-sector physical capital when α_1 increases from 0.119 to 0.15. As seen in Table 9 this is 10400000 in units of good 1. Thus, even though the value of transfer in the initial time period is zero under $\alpha_1 = 0.15$, while the same is positive when $\alpha_1 = 0.119$, the present discounted values of the stream of transfers over time is higher in the former case. Thus, significant redistribution to the informal sector from the formal sector does happen under the optimal fiscal policy even when $\alpha_1 = 0.15$.

6.2.5 Impact of changes in α_1 on the long-term growth physical capital in the formal sector, ψ_{k_1}.

To understand why the social welfare maximising value of ψ_{k_1} can be decreasing when α_1 increases, recall that

$$\psi_{k_1} = \frac{\lambda_{k_1 b} - 1}{\pi},$$

where from F3 in Section 3.1, evaluated at the optimum, the relative shadow value of formal-sector physical capital, $\lambda_{k_1 b}$, is given as

$$\lambda_{k_1 b}^* = \lambda_{k_1 b}(\boldsymbol{\tau}_k(\alpha_1), \alpha_1, \mathcal{P}) = 1 + \pi r_b - \sqrt{(1 + \pi r_b)^2 - (1 + 2\pi(1 - \boldsymbol{\tau}_k(\alpha_1))\alpha_1)}$$

Hence

$$\frac{\partial \lambda_{k_1 b}^*}{\partial \alpha_1} = \frac{\partial \lambda_{k_1 b}}{\partial \tau_k}\frac{\partial \boldsymbol{\tau}_k}{\partial \alpha_1} + \frac{\partial \lambda_{k_1 b}}{\partial \alpha_1}$$

It can be verified that $\frac{\partial \lambda_{k_1 b}}{\partial \tau_k} < 0$ and $\frac{\partial \lambda_{k_1 b}}{\partial \alpha_1} > 0$. That is, ceteris paribus, the relative shadow value of formal-sector physical capital increases as this factor becomes more productive and falls when the returns from this factor are taxed more. Further, as seen in Table 6 and discussed above, the optimal tax on physical capital increases as α_1 increases. Hence, $\frac{\partial \boldsymbol{\tau}_k}{\partial \alpha_1} > 0$. From this it follows that the sign of $\frac{\partial \lambda^*_{k_1 b}}{\partial \alpha_1}$ and hence $\frac{\partial \psi^*_{k_1}}{\partial \alpha_1}$ are indeterminate. In the context of our numerical example, as can be inferred from Table 6, $\frac{\partial \lambda^*_{k_1 b}}{\partial \alpha_1}$ and hence $\frac{\partial \psi^*_{k_1}}{\partial \alpha_1}$ take negative values. The fall in the relative shadow value of the formal-sector physical capital due to increase in the optimal tax as α_1 increases offsets the direct positive effect that increase in productivity of the resource has on its relative shadow value.

In a similar manner, we can also explain the fall in net returns to formal-sector physical capital as α_1 increases in the context of our numerical example. In particular, given that $\hat{r}^N_1 = (1 - \tau_k) r_1 = (1 - \tau_k)\alpha_1$, we have

$$\frac{\partial \hat{r}^{N^*}_1}{\partial \alpha_1} = -\alpha_1 \frac{\partial \boldsymbol{\tau}_k}{\partial \alpha_1} + (1 - \tau^*_k)$$

Thus, the sign of $\frac{\partial \hat{r}^{N^*}_1}{\partial \alpha_1}$ is ambiguous. In out numerical example, it is negative implying that the positive effect on net returns to formal sector physical capital due to increase in its productivity α_1 is more than offset by the negative effect on the same due to increase in capital taxation when α_1 increases.

6.3 Unbalanced growth.

In DM, the relevant long-run growth rates of economic variables at a macroeconomic tax equilibrium are theoretically derived for the dual economy under consideration. We concluded that, in general there will be unbalanced growth in this economy.

In the context of our numerical simulations, Tables 3 and 6 show the long-run rates of growth of various economic variables of concern employing the theory developed in DM. To see the unbalanced nature of growth in our numerically simulated economy, consider, for example, the case when productivities of unskilled labour and physical capital in the formal and informal sectors are respectively, given by $\alpha_L = 38$, $\alpha_1 = 0.119$, and $\alpha_2 = 0.1178$. Table 3 demonstrates that the long-run rate of growth of human capital is different from the long-run rate of growth of migration, n. It is greater than n and hence is determined by the relative shadow price of human capital, *i.e.*, it is given by $\psi^*_H = \frac{\lambda^*_{hb} - 1}{\theta}$.

The long-run rates of growth of both types of consumption is the same and given by $\psi^*_{\mathfrak{C}}$, which is the rate of growth of $\mathcal{C}_{11m}$ for $m = 1, 2$. The long-run rate of growth of informal-sector physical capital, ψ_{k_2} is also determined by $\psi^*_{\mathcal{C}}$, as it is bigger than the population rate of growth n. The long-run rate of growth of bond holdings, ψ_B, is given by the long-run rate of growth of human capital, which here is also equal to the rate of growth of the formal-sector physical capital.

Since the long-run rate of growth of the formal sector output is given by the greater of the long-run rates of growth of human and physical capital employed in this sector, in the case under study, it is given by either, as they are the same.

On the other hand, the long-run rate of growth of the informal sector output is given by the greater of the long-run rates of growth of unskilled labour and consumption. In our case, it is given by the latter, *i.e.,* it is $\psi_{\mathcal{C}}$.

7 Conclusions.

In this work, the theory developed in DM for modelling a contemporary dual economy and studying the features of social welfare-maximising fiscal policies in such an economy is subjected to some numerical simulations. Our numerical simulations indicate some sharp differences in the nature of solutions obtained from social welfare maximisation for human capital taxation on the one hand and physical and consumption taxation on the other. While a standard interior solution is obtained for the optimal tax on human capital, social welfare is decreasing in taxation of consumption and physical capital in the formal sector. Thus, the optimal tax rates on consumption and physical capital in the formal sector take the lowest possible values for which macroeconomic tax equilibria exist. This is zero for the consumption tax but could be positive for the tax on physical capital employed in the formal sector as there may be a positive lower bound on the set of tax rates on this form of capital that ensure that its relative shadow price and the redistributive transfer are well-defined, *i.e.,* real and non-negative.

At the social optimum obtained in our numerical simulations, there are social marginal benefits from taxing human capital to the extent that it leads to (i) reductions in socially undesirable migration, (ii) increases in socially desirable acquisition of skill by migrating labour force, (iii) increases in the tax revenue collected that is employed to finance the redistributive transfer to the informal-sector household, the public infrastructure, and the public good, and (iv) reductions in the costs (including the adjustment cost) of investment in human capital. On the other hand, there are also social marginal costs from taxation of human capital due to it – (i) reducing the relative shadow price of human capital, hence hindering investment in human capital and the long-run rate of growth of human capital and (ii) reducing its net returns, – both of which are socially desirable (*i.e.,* increase social welfare). We find that, in our numerical simulations, at the social optimum, the social marginal benefits from taxing human capital exactly offset its social marginal costs, so that the net social marginal benefit from taxing human capital is zero.

On the other hand, a universal consumption tax or a tax on physical capital in the formal sector do not affect the decisions of households pertaining to migration and acquisition of skills. The social marginal benefit of a universal consumption tax due to its positive impact on tax revenue collection, which is used for financing redistributive and infrastructural expenditures of the government, is more than offset by its social marginal cost due to the reductions in consumption that it induces for both the formal and informal sector households. Similarly, the social marginal benefits from a tax on physical capital in the formal sector due to the positive impact that it has on tax revenue and reductions in the cost of investment that it entails are more than offset by the social marginal costs of reduction in its long-run growth (and also the rate of investment) and net return. The reduction in the rate of investment is because of the fall in the relative shadow price of the physical capital due to taxation. Hence, at the social optimum, the net social marginal benefits from the consumption tax and the tax on formal-sector physical capital are negative.

In our numerical simulations, the formal-sector household is modelled to be initially better endowed as compared to the informal sector household. Social welfare maximisation using an utilitarian social welfare function leads to considerable redistribution of wealth. The optimal transfer to the informal sector ensures considerable increase in its social welfare, which is comparable to the (in fact, higher than the) social welfare of the formal sector at the social optimum implying the greater value that this inequality averse society places on the welfare of the impoverished informal sector household.

Our comparative static analyses show that reductions in productivity differential between the skilled and unskilled labour force have no impact on the optimal tax rates on consumption or the formal-sector physical capital; while reductions in the productivity differential between the physical capital employed in the formal and informal sectors have no impact on the optimal tax rates on human capital and consumption. Nonetheless, the former decreases the optimal tax rate on human capital, thereby increasing its relative shadow price and, hence, promoting its long-run growth and inducing greater extent of migration and skill formation by the migratory labour. The latter decreases the optimal tax on the formal-sector physical capital, thereby increasing its long-run growth rate and net return. At the same time, the mode of financing the redistributive transfer changes as both such productivity differentials fall: the reliance on capital taxation for financing such a transfer reduces, with more and more of the transfer and other government expenditures being financed by taxation of the economic rents/profits of the formal sector-firms that are generated by government's infrastructural expenditures in this sector. This indicates that social welfare maximising fiscal policies prefer to employ capital taxation as sparingly as possible. Rather, such policies are inclined towards employing non-capital tax instruments, whenever available, for meeting the redistributive goal in a bid to promote also the growth objective of the government.

The duality between the informal and the formal sector persists. There is unbalanced growth in these two sectors, both of which are assumed to grow endogenously in an extended AK model framework. The long-run growth of output in the formal sector is determined by the maximum of the long-run growth rates of human and physical capital employed in this sector and the growth rate of unskilled labour in the informal sector from which migration to the formal sector occurs. This will generally be higher than the long-run growth rate of output in the informal sector, which is given by the maximum of the growth rates of unskilled labour (which turns out to be the same as the rate of growth of population) and consumption. The latter is determined by the difference in the after-tax return from bond and the rate of time preference.

There is significant literature that studies the common process of structural change that many currently developed countries have undergone, where the relative importance of the agricultural sector declined and that of the manufacturing and services sectors rose over time. Many works in this literature have been comprehensively reviewed by Gabardo et al (2017). In much of this literature, the dual economy phase of structural change is seen as a mere "precursor of growth" or "a first stage of development," which the currently developed countries were soon able to leave behind. The theoretical understanding of this phase continues to be along the lines of Lewis (1954), which viewed this phase as one where the modern sector grows on account of absorbing cheap surplus labour available in the traditional sector, which is devoid of growth. In contrast, from the point of view of contemporary developing countries experiencing a demographic dividend, this is a crucial phase of development – a make it or break it point – where the future course of growth and development hinges entirely on how well the country through its institutions and policies (i) taps this dividend to achieve growth, even if unbalanced, in *both* the traditional agriculture and modern manufacturing and services sectors and (ii) can redistribute wealth so generated to promote social equity. Our study attempts to provide a framework for studying the nature of (unbalanced) growth and the types of fiscal policies that become relevant for an economy experiencing a demographic dividend.

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APPENDIX

A Solving for $\mathcal{C}_{11}(0),\ \mathcal{C}_{12}(0),\ \mathcal{C}_1(0),\ G_1$, and G_2 as functions of $s := \langle\kappa,\ \beta,\ \psi_H,\ \psi_{k_1},\ \hat{w}_H^N, \hat{r}_1^N\rangle$ and $a := \langle\tau, \alpha_L, \alpha_1, \mathcal{P}\rangle$

The macroeconomic tax equilibrium conditions for the special case of the pure formal sector are derived from the general case in DM as follows:

$$\mathcal{C}_1(0) = \left[K_2(0) + \mathcal{K}_{\mathcal{L}}L(0) - 2\mathcal{K}_{\mathcal{M}} + \frac{A_2G_2}{\alpha_2}\right]\mathcal{K}_{\mathcal{C}}{}^{-1} \tag{27}$$

$$\mathcal{C}_{11}(0) = \frac{\psi_{\mathcal{C}} - r_b}{\mathcal{Z}_9}\left[B_1(0) - \left\{\frac{\mathcal{Z}_2H_1(0)}{\psi_H - r_b} + \frac{\mathcal{Z}_7K_{11}(0)}{\psi_{k_1} - r_b} - \frac{\mathcal{Z}_{11}}{r_b}\right\}\right] \tag{28}$$

$$\mathcal{C}_1(0) = \mathcal{C}_{11}(0) + \mathcal{C}_{12}(0) \tag{29}$$

$$B(0) = \frac{\Gamma_1}{\psi_H - r_b} - \frac{\Gamma_2}{n - r_b} + \frac{\Gamma_3}{\kappa + r_b} + \frac{\Gamma_4}{\psi_{k_1} - r_b} - \frac{\mathcal{C}_1(0)\Gamma_5}{\psi_{\mathcal{C}} - r_b} + \left(\frac{2\mu_1 + G_1(\mathcal{I}A_1 - 1) + pG_2(A_2 - 1)}{r_b}\right) = 0 \tag{30}$$

$$\begin{aligned} B_2(0) = &\left\{\frac{\mathcal{Z}_2\mathcal{A}_2}{\psi_H - r_b} + \frac{\mathcal{Z}_{5a2}}{n - r_b} + \frac{\mathcal{Z}_4\mathcal{F}k_{22}(0)}{n - r_b} - \frac{\mathcal{Z}_{5b2}}{\kappa + r_b} - \frac{\mathcal{Z}_6\mathcal{E}k_{22}(0)}{\kappa + r_b} + \frac{\mathcal{Z}_7K_{12}(0)}{\psi_{k_1} - r_b}\right.\\ &+\frac{\mathcal{Z}_8k_{22}(0)\mathcal{C}_1(0)}{\psi_{\mathcal{C}} - r_b} + \frac{\mathcal{Z}_9\mathcal{C}_{12}(0)}{\psi_{\mathcal{C}} - r_b} - \left[\mathcal{Z}_{10} - \frac{r_2A_2G_2}{\alpha_2}\right]\frac{k_{22}(0)}{r_b} - \frac{\mathcal{Z}_{11}}{r_b}\\ &+\frac{\tau_kr_1K_1(0)}{\psi_{k_1} - r_b} + \frac{\tau_ww_1\mathcal{A}}{\psi_H - r_b} - \frac{\tau_ww_1\mathcal{B}}{n - r_b} - \frac{\tau_ww_1\kappa\mathcal{D}}{\kappa + r_b} + \frac{(\tau_c - g_c)\mathcal{C}_1(0)}{\psi_{\mathcal{C}} - r_b}\\ &\left.+\frac{(G_1(1 - \mathcal{I}A_1) + pG_2(1 - A_2) - 2\tau_c\mu_1)}{r_b}\right\} \end{aligned} \tag{31}$$

These can be considered as five equations that can be solved for the five variables $\mathcal{C}_{11}(0),\ \mathcal{C}_{12}(0),\ \mathcal{C}_1(0),\ G_1$, and G_2 as functions of $s := \langle\kappa,\ \beta,\ \psi_H,\ \psi_{k_1},\ \hat{w}_H^N, \hat{r}_1^N\rangle$ and $a := \langle\tau, \alpha_L, \alpha_1, \mathcal{P}\rangle$ once we note that several expressions that appear in these equations are shown to be functions of s and a in DM. This is illustrated below in points F1 through F5.

F1. The long-run growth rate of consumption, $\psi_{\mathcal{C}}$, is a fixed constant given by

$$\psi_{\mathcal{C}}(\mathcal{P}) = \frac{r_b - \rho}{\epsilon}$$

The equilibrium shadow price of human capital, λ_{hb}, is the following function of τ_w, while the the equilibrium shadow price of physical capital employed in the formal sector, λ_{k_1b}, is the following function of τ_k:

$$\begin{aligned}
\lambda_{hb} = \lambda_{hb}(\tau_w, \mathcal{P}) &= \left(1 + r_b\theta\right) - \sqrt{r_b^2\theta^2 + 2\theta r_b - 2\theta(1-\tau_w)\alpha_H} \\
\lambda_{k_1b} = \lambda_{k_1b}(\tau_k, \alpha_1, \mathcal{P}) &= 1 + \pi r_b - \sqrt{(1+\pi r_b)^2 - (1 + 2\pi(1-\tau_k)\alpha_1)} \\
\gamma = \gamma(\tau_w, \mathcal{P}) &= \frac{\lambda_{hb}(\tau_w, \mathcal{P}) - 1}{\theta} - n
\end{aligned}$$

F2. $\mathcal{K}_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}}(\tau_c, \mathcal{P}) = \frac{\eta_2(1+\tau_c)}{\eta_1 p \alpha_2}$, $\quad \mathcal{K}_{\mathcal{L}} = \mathcal{K}_{\mathcal{L}}(\alpha_L, \mathcal{P}) = \frac{\alpha_L}{\alpha_2}$, $\quad \mathcal{K}_{\mathcal{M}} = \mathcal{K}_{\mathcal{M}}(\mathcal{P}) = \frac{\mu_2}{\alpha_2}$

F3.

$$\begin{aligned}
\mathcal{E} = \mathcal{E}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \sum_m \mathcal{E}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) \\
\mathcal{F} = \mathcal{F}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \sum_m \mathcal{F}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) \\
\mathcal{A} = \mathcal{A}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \sum_m \mathcal{A}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) \\
\mathcal{B} = \mathcal{B}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \sum_m \mathcal{B}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) \\
\mathcal{D} = \mathcal{D}\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \sum_m \mathcal{D}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right)
\end{aligned}$$

where, for $m = 1, 2$:

$$\begin{aligned}
\mathcal{E}_m = \mathcal{E}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= L_m(0) - l_m(0)\frac{n}{n+\kappa} \\
\mathcal{F}_m = \mathcal{F}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= l_m(0)\frac{n}{n+\kappa} \\
\mathcal{A}_m = \mathcal{A}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= H_m(0) + \frac{\beta\kappa}{\psi_H + \kappa}\left(L_m(0) - l_m(0)\frac{n}{n+\kappa}\right) \\
&\quad + l_m(0)\frac{\beta\kappa n}{(n+\kappa)\gamma} \\
\mathcal{B}_m = \mathcal{B}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= \frac{\beta\kappa n}{(n+\kappa)\gamma} l_m(0)\left(1 - \frac{\gamma}{\psi_H + \kappa}\right) \\
\mathcal{D}_m = \mathcal{D}_m\left(\kappa, \beta, \psi_H, \psi_{k_1}, \hat{w}_H^N, \hat{r}_1^N, \tau, \alpha_L, \alpha_1, \mathcal{P}\right) &= L_m(0)\frac{\beta\kappa}{\psi_H + \kappa}
\end{aligned}$$

F4.

$$\begin{aligned}
\Gamma_1 = \Gamma_1\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \left\{\hat{w}_H^N - \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)\right\}\mathcal{A} \\
\Gamma_2 = \Gamma_2\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \Bigg[\left\{\hat{w}_H^N - \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)\right\}\mathcal{B} \\
&\quad + \frac{2\lambda_{hb}}{S'(\beta)}\left(1+\frac{S(\beta)\lambda_{hb}}{S'(\beta)}\right)\mathcal{F} - \mathcal{K}_{\mathcal{L}}(n+\delta_2)\mathcal{F}\Bigg] \\
\Gamma_3 = \Gamma_3\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \Bigg[\left\{\hat{w}_H^N - \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)\right\}\mathcal{D} + \frac{2\lambda_{hb}}{S'(\beta)}\left(1+\frac{S(\beta)\lambda_{hb}}{S'(\beta)}\right) \\
&\quad + \mathcal{K}_{\mathcal{L}}(\kappa-\delta_2)\mathcal{E}\Bigg] \\
\Gamma_4 = \Gamma_4\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \left[\hat{r}_1^N - \psi_{k_1}\left(\frac{1+\lambda_{k_1 b}}{2}\right)\right]K_1(0) \\
\Gamma_5 = \Gamma_5\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \mathcal{K}_{\mathcal{C}}(\psi_{\mathcal{C}}+\delta_2) + g_c
\end{aligned}$$

F5.

$$\begin{aligned}
\mathcal{Z}_1(\mathcal{P}) &= r_b \\
\mathcal{Z}_2 = \mathcal{Z}_2\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \left[\hat{w}_H^N - \frac{\lambda^2{}_{hb}-1}{2\theta}\right] \\
&= \left[\hat{w}_H^N - \psi_H\left(\frac{1+\lambda_{hb}}{2}\right)\right] \\
\mathcal{Z}_3 = \mathcal{Z}_3\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= w_2 - \frac{2\lambda_{hb}}{S'(\beta)}\left(1+\frac{S(\beta)\lambda_{hb}}{S'(\beta)}\right) \\
\mathcal{Z}_4 = \mathcal{Z}_4\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= (n+\delta_2-r_2)\mathcal{K}_{\mathcal{L}} \\
\mathcal{Z}_{5am} = \mathcal{Z}_{5am}\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= -\mathcal{Z}_2\mathcal{B}_m + \mathcal{Z}_3\mathcal{F}_m \\
\mathcal{Z}_{5bm} = \mathcal{Z}_{5bm}\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= -\mathcal{Z}_2\mathcal{D}_m + \mathcal{Z}_3\mathcal{E}_m \\
\mathcal{Z}_6 = \mathcal{Z}_6\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= (\delta_2-\kappa-r_2)\mathcal{K}_{\mathcal{L}} \\
\mathcal{Z}_7 = \mathcal{Z}_7\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= \left[\hat{r}_1^N - \psi_{k_1}\left(\frac{1+\lambda_{k_1 b}}{2}\right)\right] \\
\mathcal{Z}_8 = \mathcal{Z}_8\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= -(\psi_{\mathcal{C}}+\delta_2-r_2)\mathcal{K}_{\mathcal{C}} \\
\mathcal{Z}_9 = \mathcal{Z}_9\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= -(1+\tau_c)(1+\frac{\eta_2}{\eta_1}) \\
\mathcal{Z}_{10}(\mathcal{P}) &= 2r_2\mathcal{K}_{\mathcal{M}} \\
\mathcal{Z}_{11} = \mathcal{Z}_{11}\left(\kappa,\beta,\psi_H,\psi_{k_1},\hat{w}_H^N,\hat{r}_1^N,\tau,\alpha_L,\alpha_1,\mathcal{P}\right) &= (1+\tau_c)\mu_1 - \frac{\mu_2[r_b+\delta_2]}{\alpha_2}
\end{aligned}$$