# Discussion Papers in Economics

## Impact of Electoral Competition, Swing Voters and Interest Group Lobbying on Strategic Determination of Equilibrium Policy Platforms

**Deepti Kohli and Meeta Keswani Mehra**

Discussion Paper 20-02


Centre for International Trade and Development
School of International Studies
Jawaharlal Nehru University
India

# Impact of Electoral Competition, Swing Voters and Interest Group Lobbying on Strategic Determination of Equilibrium Policy Platforms

Deepti Kohli[1] Meeta Keswani Mehra[2]

**Abstract**

We extend the probabilistic voting model of Persson and Tabellini (2002) by utilizing exogenous parameters to capture corruption and the effectiveness of campaign spending expenditure incurred by purely opportunistic electoral candidates. Incorporation of ideological differences amongst voters as well as the embezzlement of campaign funds received by the electoral candidates from the interest groups gives rise to a dual uncertainty within the model. We derive the equilibrium policy positions of the two opportunistic candidates in the scenario where none of the above uncertainties exist (the benchmark case), where only uncertainty about voters' preferences exist (swing voter case), and where both these uncertainties exist (case where both swing voters and lobby groups exist). We also provide a detailed comparison of the policy choices across these three equilibrium specifications. Furthermore, our comparative statics findings indicate that the impact of a change in various parameters, such as, the difference between the payoff received from winning and losing an election, a political candidate's corruption parameter, the proportion of uninformed voters and the ideological density of a voter group, on an electoral candidate's equilibrium policy platform relies on two main strategic forces, apart from the inherent centripetal effect and the influence of campaign fund embezzlement, namely, the relative swing voter effect and the relative organizational strength of lobbies effect. In sum, the equilibrium tax platform is found to sway in favour of the more dominant effect and towards the economic policy preferences of the voter group corresponding to the relatively stronger effect.

*JEL Classifications:* D72, D73, H41, P16
*Keywords:* Electoral Competition, Swing Voters, Interest Groups, Corruption

[1]Research Scholar, Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi-110067. Email:kohli.deepti@rediffmail.com

[2]Professor of Economics, Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi-110067. Email:meetakm@mail.jnu.ac.in

# 1 Introduction

The analysis of theoretical models of electoral competition ultimately leads to a discussion on equilibrium policy platforms of the electoral candidates in question. Candidates during electoral competition are expected to move their policy platforms so as to maximize their vote-share, and hence, their chances of winning the election. According to Downs (1957), when two electoral candidates are purely opportunistic; a candidate located, say, to the left of her competitor always has the incentive to approach her competitor's location because all voters with ideal policies to her left will always vote for her (centripetal force). Thus, this player unambiguously raises her vote-share by approaching her competitor's policy position. This holds true for both the candidates, and hence, there occurs complete convergence to the median voter policy position in the Downsian model of electoral competition.

However, the belief that the political process only serves the interests of the median voter does not ring true under a real-world scenario. For instance, the policy convergence prediction does not hold true for the U.S., a country that was supposed to be political benchmark for Down's work on this topic (Frendreis et al., 2003; Stonecash et al., 2003; Grofman, 2004). In addition, the empirical evidence as provided by studies like Ansolabehere et al. 2001; Gerber and Lewis, 2004, etc. also does not support the Downsian predictions. For instance, studies like McCarty et al. (2006) and Bruter and Harrison (2007) respectively assert that the electoral parties (and their candidates) in the U.S. and U.K. are predominantly becoming more polarized with time. Theoretically, several studies have recognized another opposing force at work, that is, the centrifugal force which helps explain the incomplete policy convergence and/or policy divergence phenomenon which is absent in the standard Downsian model. This centrifugal force can take the form of policy and ideological preferences of voters, role of special interest groups as well as the electoral objectives of candidates.

For a complete analysis of equilibrium policy outcomes under electoral competition, one cannot ignore the strategic interactions between policymakers (or political candidates) and interest groups. Lobbies or special interest groups are said to have a crucial role in the elections because of their contribution to political parties, endorsement of electoral candidates, and provision of information to the public. Presence of lobbies engenders an opposing force at work, that is, the centrifugal force (coined by Cox (1990)) which helps explain the incomplete policy convergence and/or policy divergence phenomenon. This happens if, electoral platforms systematically favor certain organized groups, those groups will also adapt their campaign contributions accordingly. Consequently, the political parties or the representative candidate will have to settle the trade-off between gaining more votes by pushing policy towards the median on one hand, and, securing more monetary contributions by swaying policy towards the interest group's preferences on the other.

In the field of political economy, a vast body of literature is present on models which combine

both electoral competition and lobbying activities. Austen-Smith (1987) develops a simple model to analyze the impact of campaign contributions on electoral-policy decisions of candidates where interest groups are firms that select contributions pertaining to the assumption that candidates' policies and opposing groups' donations remain unchanged. Candidates utilize campaign contributions to affect policy-oriented voters' perceptions of candidates' positions. The author deduces that in this framework the introduction of campaign contributions may impact candidates' electoral policies, in which case exactly one of the two interest groups are at an advantage. Baron (1994) develops a model of electoral competition in which candidates collect campaign contributions by selecting policies that benefit interest groups and then utilize those contributions to influence voters who are uninformed about the policies. Informed voters, however, vote based on those policies, so candidates face a trade-off between choosing a policy to generate funds to attract the uninformed vote and choosing a policy to attract the informed vote. In this study, electoral equilibria are characterized by two categories of policies: particularistic and collective. In the case of particularistic policies, the equilibrium policies of the candidates are separated if the proportion of uninformed voters is sufficiently high, and the degree of separation is an increasing function of that proportion. In contrast, in case of collective policies, the candidates locate at the median of the ideal points of the informed voters, and contributions are zero.

A pioneering work in this field is by Grossman and Helpman (1996), which explores the Downsian model of electoral competition where candidates select policies which help maximize their chances of winning the elections. In this common agency setting, lobbying thus induces candidates to choose policies that are a compromise between the policy preferences of voters and the lobbies. Persson and Tabellini (2002) develop a simple probabilistic voting model of electoral competition with exogenously given organized interest groups. They highlight the importance of swing voters, who become more influential in the electoral campaign, and both candidates seek to please them, rather than the electorate at large. Moreover, they assert that campaign contributions matter as they allow politicians to increase their relative popularity in the electorate at large. Groups organized to provide such contributions are expected to become more influential in the electorate campaign and receive policy favors at the expense of the unorganized. In yet another paper, Grossman and Helpman (2005) present a novel model of campaigns, elections, and policy-making in which the ex-ante objectives of national party leaders differ from the ex-post objectives of elected legislators. This generates a distinction between "policy rhetoric" and "policy reality" and introduces an important role for "party discipline" in the policy-making process, thereby indicating a protectionist bias in majoritarian politics. In slightly different contexts, a few papers addressing the issue of campaign contributions affecting electoral outcomes include studies by, Coughlin et al. (1990), Morton and Myerson (1992), Mayer and Li (1994), Groseclose and Snyder (1996), Besley and Coate (1997, 2001), Persson and Helpman (1998), Prat (2002) among

others.

Another aspect of political economy literature is cognate with the role of political corruption in electoral competition models. It is believed that political corruption is often closely linked to the lobbying activities of special interest groups. For instance, political contribution given to electoral candidates by lobbies may be directed towards either personal gain or political purposes, rather than for campaigning on the issue intended by the interest group. This is highly probable when a donor has no credible means of retaliation, or when the political party's grip on power is uncertain. The link between interest groups and political corruption and the role it plays in electoral competition models has been analyzed by a few studies. For instance, Damania and Yalcin (2008) examine the nature of the interaction between the lobbying activities of special interest groups and the occurrence of political corruption and determine if electoral competition can eliminate political corruption. They assert that higher electoral competition works towards reducing policy distortions, which however, encourages more intense lobbying which in turn increases the scope of corrupt behavior. Le and Yalcin (2018) research the impact of lobby groups on electoral competition and equilibrium policy outcomes by utilizing a "money for policy favours" model of lobbying. They show that, in case of embezzlement of campaign funds, political parties that divert more funds for personal gain stand on more independent platforms and necessitate increased contributions from lobby groups. In essence, even though greater electoral competition leads to lower policy distortions, this, in turn, spurs more intense lobbying, thereby increasing the scope of misappropriation of campaign funds. In a slightly different context, Wilson and Damania (2005) investigate the effect of corruption on environmental policy under varying degrees of political competition where a polluting firm strives to lower the amount of environmental tax it pays by bribing a low level bureaucrat to make false reports regarding emission levels. Their findings indicate that higher levels of political competition lead to stricter regulations and better environmental outcomes and that political competition has the capacity to decrease both grand and petty corruption, though this is not guaranteed.

As is apparent from the above discussion, the political economy of elections inherently involves a great deal of interaction between three separate components: economic and ideological preferences of voters, motivation of special interest groups and electoral objectives of candidates. In this paper, we illustrate how electorally motivated lobbying may influence policy and explore the different strategic forces at work, namely the swing voter effect and the relative organizational strength of lobbies effect, apart from the conventional centripetal force (traditionally known as the median-voter effect), which shape the policy choices of competing political candidates. Our basic model follows the combined lobbying and probabilistic voting model of Persson and Tabellini (2002) by utilizing exogenous parameters to capture corruption and effectiveness of campaign spending expenditure by electoral candidates. The probabilistic nature of the model helps in

focusing on the ideological differences across voters and, hence, segregate or tease out the impact of swing voters on electoral outcomes. Another key feature embedded in this model is that the electoral candidates embezzle a portion of funds they receive from interest groups. Both these attributes create a dual uncertainty within the model. The first is related to the random factors that can potentially affect voter's decisions to vote for a certain candidate, which entails that electoral candidates in our model have incomplete information about voter's preferences. The second uncertainty arises on part of lobby groups who are unsure whether an electoral candidate will honestly utilize their contributions to increase their chances of electoral success. We derive the equilibrium policy positions of the two opportunistic candidates in the scenario where none of the above uncertainties exist (the benchmark case), where only uncertainty about voter's preferences exist (swing voter case), and where both these uncertainties exist (case where both swing voters and lobby groups exist). In general, we go beyond the benchmark Persson and Tabellini (2002) framework by providing a comparison of the equilibrium policy platforms of the political candidates across the above mentioned underlying specifications of the theoretical framework. To the best of our knowledge, an explicit comparison of different policy equilibria across distinct specifications of the model (as mentioned above) has not been attempted in any related research so far, and this constitutes a novelty of this analysis.

In particular, a comparison of tax platforms of electoral candidates across the three distinct equilibrium specifications reveals that an opportunistic candidate's tax platform in the swing voter case is always lower than the tax platform of the same candidate in the benchmark case. This is because, in the benchmark case the ideological density across voter groups is taken to be invariant and, hence, the electoral candidate assigns equal weights (a value of 1) to each voter group since each voter is perceived to be the same by her. However, in the swing voter case, the candidate assigns a higher weight to the group that has a larger number of swing voters (or more ideologically homogeneous population) and a lower weight to the groups with a lesser number of swing voters (or with more ideologically heterogeneous population) to increase her chances of winning the election.[3] Thus, the presence of swing voters in the economy effectively reduces the level of public good provision as compared to the public good provision under the benchmark scenario. Furthermore, the level of public good provision (and the associated tax policy) will be higher in the case that involves both swing voters and interest groups as compared to the benchmark case

[3]As explained later in Lemma 1 and result $R1$ of this paper, the ideological densities within a voter group symbolize how voters in each group respond to deviations in economic policy. Greater ideological density within a group implies more number of ideologically indifferent or non-partisan voters and, hence, more number of (swing) voters who can be swayed through slight policy deviations by the office-seeking political candidates. In contrast, a lower ideological density within a voter group represents greater heterogeneity amidst the overall range of the voters' ideological bias and, therefore, more number of (non-swing) voters who have more firm ideological opinions and cannot be swayed by perfunctory policy deviations undertaken by opportunistic electoral candidates.

if both organizational strength as well as ideological density of a voter group favouring a higher provision of public good is greater than the organizational strength and ideological density of the voter group favouring a lower public good provision.

In addition to this, a comparison of the equilibrium tax platform in the presence of both interest groups and voter groups having differing ideological densities with the equilibrium tax platform under the swing voter case illustrates that the tax platform under the former scenario will be higher (lower) than the tax platform under the latter scenario if the preference for public good provision of both the voter groups is greater (lower) than the actual public good provision under the swing voter case. In other words, if the public good provision under the swing voter case is low (high) in relation to what the voters in the economy as a whole prefer, then the introduction of lobby groups in this scenario would cause a rise (fall) in the tax platform of electoral candidates. Accordingly, the provision of public good would also increase (decrease). In contrast, if one voter group in the economy prefers more public good relative to the actual level of provision while the other group prefers relatively less of it, then the introduction of two such antithetical interest groups in the swing voter case would create contradictory forces at work, where, one interest group will lobby for an increase in the public good provision, while the other will lobby for a reduction in the level of public good provision. Hence, the equilibrium tax choices of electoral candidates in the presence of swing voters and opposing lobby groups will adjust accordingly to whichever of the two lobby groups is stronger in terms of its organizational strength.

Furthermore, this paper makes a specific contribution to the existing literature through the comparative statics results based on how campaign contributions respond to changes in the individual political and economic parameters of the model (example, honesty, difference between payoff from winning and losing an election, popularity shock, effectiveness of campaign expenditure, etc). Specifically, the comparative statics show that as the electoral candidate becomes more honest with respect to the spending of the contribution money for campaigning (that is, indulges in lesser leakage of monetary funds), it induces the individual members of the supporting interest group to raise the level of donations to that candidate. This is because, donation money is now used by an electoral candidate for the purpose of campaign spending, which in turn, is used to influence voters to win elections. If the political candidate does not indulge in greater leakage of campaign funds, the marginal benefit derived by the individual donor would be greater due to smaller leakage of money, which in turn incentivizes the member to contribute more. We also find that an increase in the organizational ability of a voter group leads to two opposite impacts on individual campaign donations by the same group. On the one hand, the electoral candidates try to sway the uninformed voters in the economy through campaign spending, and, if the proportion of such uninformed voters is more relative to the proportion of individuals who are a part of a lobby, an increase in the organizational strength of the lobby results in greater contributions being

offered to the electoral candidate at an individual level. This is the uninformed voter effect. On the other hand, there is the direct organizational strength effect of lobby, which suggests that if a lobby is already strong in terms of its organizational capability relative to the proportion of uninformed voters in the economy, then the lobby members need not put too much effort into providing electoral candidates with greater amounts of donation money. Therefore, the impact of a rise in a lobby group's organizational strength on the campaign donations is ambiguous and depends on which of the above two effects dominate.

Finally, we derive the comparative statics of equilibrium tax platforms with respect to other important parameters such as difference between the payoff received with winning and losing an election, effectiveness of campaign spending, a political candidate's corruption (or leakage) parameter, policy preference parameter, the popularity shock parameter, the proportion of people influenced by campaign spending and the ideological density of a voter group. For each of these, we find that the electoral policy platforms of corrupt political representative are influenced through the following key channels: the relative swing voter effect, the relative organizational strength of lobbies effect and the inherent median voter effect (in this model the mean voter effect or the centripetal effect). In sum, we conclude that the equilibrium policy (tax) platform of the electoral candidates sways in favour of the effect that is relatively more powerful and towards the policy preferences of the voter group corresponding to the relatively stronger effect. The framework of the model is illustrated in detail in the next section.

The rest of the paper is organized as follows. Section 2 outlines the basic structure of the model and the stages of the game. Section 3 lays out the characterization of equilibrium for different specifications of the model. This is followed by a comparison of policy platforms in different equilibrium specifications of the model in Section 4. Section 5 deals with the comparative statics and provides intuitive explanation for the derived results. Finally, Section 6 concludes.

## 2 Theoretical Framework

The analytical framework of electoral competition and special interest politics in this paper is based on the probabilistic voting model of Persson and Tabellini (2002), which characterizes electoral competition between two opportunistic candidates. Their model is extended to incorporate the dishonest (or corrupt) nature of political candidates along with the presence of exogenously given lobby groups that comprise of citizen-voters who can influence policies through campaign contributions. The following sub-sections discuss the outline of the theoretical model.

## 2.1 Consumption, Production and Government

The basic framework is derived from Redoano (2010). There is an economy with a population of size $n$. Residents consume a private good $c$ and a local public good $g$. Output $y$ is produced from labour, which is inelastically supplied by each individual in an amount equal to unity. The production technology is assumed to be linear in aggregate labour input, and by suitable choice of units, the wage rate $w$ is normalized to unity. Output $y$ is used for private consumption and for the provision of the public good $g$. The MRT (marginal rate of transformation) between the private consumption good and public good in production is assumed to be unity. The provision of the public good is funded by a lumpsum income tax levied on every individual at a common rate $t$, and the government budget constraint is $g = tn$. Accordingly, private good consumption for an individual is $c = w - t$, where with $w = 1$, $c = (1 - t)$ and public good provision is $g = tn$. From this, it can be shown that $y = n$ as follows:

$$y = nc + g,$$

$$\implies y = nc + nt \iff y = n(c + t).$$

Using the fact that $c = (1 - t)$, it is easy to show that in equilibrium,

$$y = n. \tag{1}$$

The above expression implies that, based on the assumed production technology, the aggregate production in our stylized economy equals total population. This is because one individual in the economy is endowed with one unit of labour and, hence, produces one unit of output given that production technology is linear in aggregate labour input.

## 2.2 Electoral Candidates

There are two opportunistic political candidates, $X$ and $Y$, who engage in electoral competition. The political candidates are dishonest in the sense that when they receive campaign contributions from lobbies, they spend only a fraction of that money on voters in the form of campaign advertisements and keep the rest for private use. The fraction of money spent on voters by candidate $X$ is denoted by $\beta_X$ and that by candidate $Y$ is denoted by $\beta_Y$. In this sense, $\beta_X$ and $\beta_Y$ represent honesty or corruption parameters. We assume that $\beta_X$ and $\beta_Y \in [0, 1]$. In particular, $\beta_K = 0$ represents no expenditure on campaign advertisements being incurred by the $K$th electoral candidate, while $\beta_K = 1$ implies that the entire amount of campaign contributions are spent on campaign advertisements by candidate $K$, where $K = X, Y$. In other words, $\beta_K = 0$ represents a completely dishonest political candidate, whereas $\beta_K = 1$ represents a completely honest electoral candidate, where $K = X, Y$. Moreover, when $\beta_K = 0.5$, this means that the $K$th electoral candidate spends half of the contribution money on campaign advertisements and keeps

the other half for private use, that is, now candidate $K$ (where $K = X, Y$) is perceived as being neither honest nor corrupt. The spending on campaign advertisements is assumed to enhance the popularity of the individual political candidates amongst voters. Moreover, if a candidate (say $X$) wins the election, she receives a payoff $R$ but if she loses, she receives $Q$, where, $R > Q$. Therefore, candidate $X$ maximizes the following objective function:

$$p_X[(1-\beta_X)C^X + R] + (1-p_X)[(1-\beta_X)C^X + Q]. \tag{2}$$

where, $p_X$ is the probability of winning of candidate $X$, $(1-p_X)$ is the probability of winning of candidate $Y$, $(1-\beta_X)$ is the proportion of contributions kept for private use by candidate $X$ and $C^X$ is the aggregate campaign contributions received by candidate $X$.

### 2.3 Voters

Citizens are divided into two different types on the basis of their preference for economic policy (or public good), namely, Low preference type ($L$) and High preference type ($H$). Their policy bliss points are denoted by the parameter $\theta_j$, where, $\theta_j \,\epsilon\, \{\theta_L, \theta_H\}$; and, $\theta_L < \theta_H$. Further, the population in each voter group, $j\epsilon\{L, H\}$, is denoted by $n_j$ and the share of population in each group is $\eta_j$, where, $\eta_j = \frac{n_j}{n}$. These population parameters are assumed to be given exogenously in the model. Following Redoano (2010), we assume that each citizen (or voter) of type $j$ has quasi-linear preferences over private consumption:

$$u_j(g) = c_j - \frac{1}{2}\left(g - \theta_j - \frac{1}{n}\right)^2; \quad \theta_j \,\epsilon\, \mathbb{R}. \tag{3}$$

Citizens with higher $\theta$'s have higher valuations of the public good:

$$u_j(t) = (1-t) - \frac{1}{2}\left(tn - \theta_j - \frac{1}{n}\right)^2; \quad \theta_j \,\epsilon\, \mathbb{R}. \tag{4}$$

It can be easily seen that maximization of equation (4) results in the following first-order condition

$$\frac{\partial u_j(t)}{\partial t} = -1 - n^2 t + n\theta_j + 1 = 0,$$

which in turn results in the following equilibrium tax platform:

$$t = \frac{\theta_j}{n}. \tag{5}$$

This represents the first-best solution for tax policy where each individual in the economy pays tax in proportion to $\theta_j$, that is, their most preferred level of public good provision.

The term $\frac{1}{n}$ in the utility function represents adjustment of the bliss point of a voter in group $j$. For instance, a higher $n$ implies a lower adjustment of $\theta_j$ and a lower value of $n$ indicates a higher adjustment of $\theta_j$. This means that, for any citizen-voter of type $j$ in the economy, the disutility due to a deviation of the actual level of public good provision from her bliss point ($\theta_j$) is

enhanced with the presence of the term $\frac{1}{n}$. Thus, we can conclude that, for extremely large-sized economies (that is, when $n$ tends to a very large value) the deviation in terms of disutility of an individual is not significantly affected by the population size, or it can be stated that it changes by a relatively small marginal amount. In contrast, for small-sized economies (that is, when $n$ takes on a very small value), the deviation in terms of disutility of an individual is markedly affected by the population size, or it can be stated that it changes by a relatively larger marginal amount.

Apart from this, voters in each group can also differ along another dimension that is not related to economic policy variable, $t$, which will be referred to as the ideological bias from hereon. Using the probabilistic voting paradigm, this ideological dimension cannot be influenced by the choice of the electoral policy platform. In this model, $\sigma_j^i$ denotes the ideological bias of a voter $i$ in group $j$, and it has group-specific uniform distribution on the domain $\left[\frac{-1}{2\phi_j}, \frac{1}{2\phi_j}\right]$, where $\phi_j$ is the ideological density of group $j$ and each group has members inherently biased towards one or the other candidate. Moreover, when $\sigma_j^i = 0$, the voter is considered to be ideologically neutral. Without loss of generality, we assume that when $\sigma_j^i < 0$, the voter is closer to candidate $X$ and when $\sigma_j^i > 0$, the voter is closer to candidate $Y$. Further, since $\phi_j$ is ideological density of a group, $\phi_j \in [0, 1]$. In this respect, the structure of our model is similar to Persson and Tabellini (2002) in an opportunistic modeling framework.

Additionally, a parameter $\widetilde{\alpha}$ represents candidate $Y$'s average (relative) popularity in the population as a whole before elections and it is assumed to follow a uniform distribution over the range $\left[\frac{-1}{2\psi}, \frac{1}{2\psi}\right]$. This popularity parameter constitutes certain characteristics of electoral candidates, as perceived by voters such as the existing (positive or negative) overall image of an electoral candidate in the minds of the voters as well as their public leanings in respect of electoral and fiscal accountability. It should be noted that $\widetilde{\alpha}$ can be positive or negative. If $\widetilde{\alpha} > 0$, then candidate $Y$ is assumed to be relatively more popular than $X$, and if $\widetilde{\alpha} < 0$, then candidate $X$ is assumed to be more popular relative to $Y$.

### 2.4 Interest Groups

We do not model the lobby formation process in this paper and assume that all lobby groups are formed exogenously. It is assumed that an exogenously given proportion of citizens of type $L$ and $H$ get organized to form their respective lobbies and offer contributions for campaign expenditure to the two electoral candidates and attempt to sway the political candidates towards their respective policy bliss points in terms of the amount of public good provision in the economy. Since the level of public good provision depends upon the amount of tax imposed on the citizens (given balanced budget), these lobbies indirectly try to influence the electoral candidate $X$'s and $Y$'s tax platforms, $t_X$ and $t_Y$, respectively. But lobbies (as citizens) bear the cost of this tax as well, so, two opposing effects are at work here: one, benefit from preferred public good provision; and two,

adverse effect of tax imposition.

The interest groups in this model have an influence motive for donating to the electoral candidates, that is, lobbies are only concerned about the policy which would be implemented and not about who wins the election. More specifically, if a group of voters organize themselves as an interest group $j$, where $j \epsilon \{L, H\}$, then they are able to contribute to political candidates. This, in turn, is used to influence voters via campaign spending. It should also be mentioned here that while solving the model, it is assumed that the lobbies concentrate their efforts on securing a policy (tax) outcome (on which their interests are aligned) that is to the group's liking, while ignoring the ideological and/or popularity outcomes (on which their interests may not be aligned). Voters from the organized class are assumed to be immune to campaign spending while, if a voter is unorganized, overall campaign spending by an electoral candidate will affect the voter's perceived popularity of that candidate in a way that is linear with respect to the difference between candidate $X$ and candidate $Y$'s total spending. Specifically, we use $O_j \in [0, 1]$ as a parameter to denote the organizational strength of the lobby groups, where,

$$O_j = 1, \text{if voter group j is completely organized,}$$

$$O_j = 0, \text{ if voter group j is completely unorganized.}$$

### 2.5 Stages of the Game

Solution to the above model assumes a game-structure. The stages of the game are as follows:
1) *Policy Announcement Stage*: Two candidates, $X$ and $Y$, simultaneously announce their electoral policy platforms, $t_X$ and $t_Y$, respectively.
2) *Lobbying Stage*: Lobbies offer (monetary) contributions to the electoral candidate they favour in order to move their policy choice towards the lobby's preferred choice in terms of public good provision.
3) *Voting Stage*: Stochastic factors that affect voters' preferences about electoral candidates, that is, $\sigma_j^i$ and $\widetilde{\alpha}$ ($j \in \{L, H\}$ and $i = 1, ..., n$), are realized and all uncertainty is resolved. Elections are held and voters vote for one of the two political candidates.

The model is solved using backward-induction. Equilibria for the three different specifications of electoral competition are characterized, which are explained as follows:

*Case 1: The Benchmark Case*
In the benchmark case, we assume that no special interest groups or lobbies exist and the ideological density is uniform across all the groups of voters, that is, $\phi_j = \widetilde{\phi}$, where $j \in \{L, H\}$.

*Case 2: The Swing Voter Case*
In this case, again no lobbies exist but the ideological density, $\phi_j$ ($j \in \{L, H\}$) is not uniform

across all groups of voters. This indicates the presence of swing voters in each voter group, and the group which has a higher ideological density signifies a greater proportion of swing voters in that group relative to the other group, thereby making it more attractive from an electoral candidate's point of view. As elucidated in Persson and Tabellini (2002), greater ideological density within a group implies more number of ideologically indifferent or non-partisan voters and, hence, more number of (swing) voters who can be swayed through slight policy deviations by the office-seeking political candidates. In contrast, a lower ideological density within a voter group represents greater heterogeneity amidst the overall range of the voters' ideological bias and, therefore, more number of (non-swing) voters who have firm ideological opinions and cannot be swayed by perfunctory policy deviations undertaken by opportunistic electoral candidates.

*Case 3: Electoral Competition with Interest Groups and Swing Voters*
In this case, besides the voter groups having differing ideological densities, the interest groups or lobbies are also present who influence the electoral candidates with their campaign donations.

Since the first two cases are special cases of the third one, we first solve for the third case and then derive the policy equilibria for the first two cases by utilizing its outcome.

# 3 Characterizing the Equilibria

To solve the model, we start with the final stage of the game in which $\widetilde{\alpha}$ and $\sigma_j^i$ are realized. The last stage comprises of a probabilistic voting setting in which the candidates are uncertain about voter's preferences. Thus, given the policy platforms, $t_X$ and $t_Y$, of the two political candidates, a voter $i$ in group $j$ would vote for candidate $X$ if

$$u_j(t_X) > u_j(t_Y) + \sigma_j^i + \alpha.$$

And similarly, a voter $i$ in group $j$ would vote for the candidate $Y$ if

$$u_j(t_X) < u_j(t_Y) + \sigma_j^i + \alpha.$$

where,

$$\alpha = \widetilde{\alpha} + h(1 - O_j)[\beta_Y C^Y - \beta_X C^X]. \tag{6}$$

The last expression, $\alpha$, measures the popularity of candidate $X$ relative to candidate $Y$ and this expression comprises of a stochastic element $\widetilde{\alpha}$, as well as the difference between campaign spending of the two candidates, which can be used to influence those voters who are not organized as interest groups, and where, $h$ denotes the effectiveness of campaign spending.

Furthermore, if

$$u_j(t_X) = u_j(t_Y) + \sigma_j^i + \alpha,$$

then, the voter $i$ in group $j$, after considering the policy platforms and average popularity of the two political candidates, is perceived to be indifferent between voting for $X$ or $Y$. Such voters are conventionally known as swing voters. The swing voters in voter group $j$ can be defined as:

$$\sigma_j = u_j(t_X) - u_j(t_Y) - \alpha. \tag{7}$$

From equation (7), it can be inferred that everybody with $\sigma_j^i < \sigma_j$ will vote for candidate $X$, while everybody with $\sigma_j^i > \sigma_j$ will vote for candidate $Y$. This brings us to the following lemma.

Lemma 1: *If both the electoral candidates, $X$ and $Y$, choose the same (tax) policy platform, then the swing voter in group $j$ will be represented by the following expression:*

$$\sigma_j = -\alpha,$$

*provided that the voters from both groups have identical utility functions.*

If both political candidates choose the same tax platform, then the utility derived by a voter $j$ when either candidate $X$ or candidate $Y$ wins will be identical, that is, $u_j(t_X) = u_j(t_Y)$. Therefore, equation (7) reduces to the following expression:

$$\sigma_j = -\alpha.$$

This result explicitly shows the significance of a swing voter in our electoral framework. A swing voter is relevant because a slight deviation in the policy platform is adequate to gain her vote in the probabilistic voting Nash equilibrium, where the office-seeking political contenders only care about winning the election. In other words, given the presence of swing voters in each group, an electoral candidate will now compare her gain or loss in vote share from each group on account of a unilateral deviation from the initial equilibrium platform. This gain or loss of votes will depend on the number of swing voters in each voter group, thereby making these swing voters an essential determinant of the win probability of an electoral candidate in a probabilistic setting.

Accordingly, the vote share of candidate $X$ in group $j$ can be expressed as:

$$\pi_j^X = \phi_j \left( \sigma_j + \frac{1}{2\phi_j} \right).$$

Or, using equation (7), it can be written that,

$$\pi_j^X(t_X, t_Y) = \frac{1}{2} + \phi_j[u_j(t_X) - u_j(t_Y) + h(1 - O_j)(\beta_X C^X - \beta_Y C^Y) - \widetilde{\alpha}].$$

Candidate $X$'s aggregate vote share can be found by summing up the above expression across $j$. So, we have,

$$\pi^X(t_X, t_Y) = \sum_j \eta_j . \pi_j^X(t_X, t_Y),$$

which implies that,

$$\pi^X(t_X, t_Y) = \frac{1}{2} + \sum_j \eta_j.\phi_j[u_j(t_X) - u_j(t_Y) + h(1 - O_j)(\beta_X C^X - \beta_Y C^Y)] - \widetilde{\alpha}\phi.$$

where, $\phi = \sum_j \eta_j.\phi_j$ is the average or mean ideology of the entire population.
Or,

$$\pi^X(t_X, t_Y) = \frac{1}{2} + [u(t_X) - u(t_Y) + h\delta(\beta_X C^X - \beta_Y C^Y)] - \widetilde{\alpha}\phi, \tag{8}$$

where, $u(t_X) = \sum_j \eta_j.\phi_j u_j(t_X)$, $u(t_Y) = \sum_j \eta_j.\phi_j u_j(t_Y)$ and $\delta = \sum_j \eta_j.\phi_j(1 - O_j)$ represents the proportion of population that is not organized as an interest group and is, therefore, influenced by campaign expenditure. We can think of voters not organized as akin to uninformed voters who can be influenced by political candidates through higher campaign spending, while those who manage to organize themselves can be termed as informed voters on whom there is no effect of political candidate's campaign spending. Candidate $X$ will win when $\pi^X(t_X, t_Y) > \frac{1}{2}$, which implies that,

$$\widetilde{\alpha} < \frac{[u(t_X) - u(t_Y) + h\delta(\beta_X C^X - \beta_Y C^Y)]}{\phi} = \underline{\widetilde{\alpha}}(t_X, t_Y),$$

where, $\underline{\widetilde{\alpha}}(t_X, t_Y)$ is some threshold level of popularity. This brings us to our next lemma which is stated as follows:

*Lemma 2: Given the respective choice of (tax) policy platforms of electoral candidates $X$ and $Y$, that is, $t_X$ and $t_Y$, political candidate $X$ will win the election if $\widetilde{\alpha} < \underline{\widetilde{\alpha}}(t_X, t_Y)$ and political candidate $Y$ will win the election if $\widetilde{\alpha} > \underline{\widetilde{\alpha}}(t_X, t_Y)$.*

The threshold popularity parameter level $\underline{\widetilde{\alpha}}(t_X, t_Y)$ depends on some deterministic factors such as choice of policy platforms of both the electoral candidates as well as their campaign spending, which are crucial in shaping the popularity and, hence, their probability of win. But, the popularity of any political candidate also has an inherent stochastic element which cannot be correctly ascertained. This result shows that, when the value of that stochastic popularity shock component ($\widetilde{\alpha}$) is realized and is found to be lower than the cut-off value $\underline{\widetilde{\alpha}}(t_X, t_Y)$, it signifies a positive shock for candidate $X$'s popularity just before the election, which helps her win the election by transferring a higher number of voters in her favour. In contrast, when the random popularity shock parameter realizes a value exceeding the threshold level $\underline{\widetilde{\alpha}}(t_X, t_Y)$, candidate $Y$ receives a positive popularity shock just before the election, in turn increasing $Y$'s probability of winning.

Using Lemma 2, we can state that, since the candidates do not know $\alpha$, they will set the policy platform to maximize the probability of winning the election as:

$$Pr\left[\pi^X(t_X, t_Y) > \frac{1}{2}\right] = Pr[\alpha < \underline{\widetilde{\alpha}}(t_X, t_Y)] = \frac{1}{2} + \psi\underline{\widetilde{\alpha}}(t_X, t_Y),$$

$$\iff Pr\left[\pi^X(t_X, t_Y) > \frac{1}{2}\right] = p_X(t_X, t_Y) = \frac{1}{2} + \frac{\psi[u(t_X) - u(t_Y) + h\delta(\beta_X C^X - \beta_Y C^Y)]}{\phi}. \tag{9}$$

And, since candidate $Y$ wins with probability $p_Y(t_X, t_Y) = (1 - p_X(t_X, t_Y))$, we have,

$$p_Y(t_X, t_Y) = \frac{1}{2} - \frac{\psi[u(t_X) - u(t_Y) + h\delta(\beta_X C^X - \beta_Y C^Y)]}{\phi}. \tag{10}$$

These probabilities form the basis for setting up the objective functions of the political candidates. It can be seen from the above that, as both individual utility functions are continuous functions of policy choice, $t_X$ and $t_Y$, the probability of winning also becomes a continuous function of the distance between the two electoral platforms. Since these probabilities depend on the amount of total campaign spending by the candidates, we need to focus on the next stage of lobbying that help determine the aggregate contributions to individual political candidates. But, before solving for contributions by interest groups, we characterize the benchmark and the swing voter case, since no lobbies are present in both cases, thus entailing zero campaign contributions.

### 3.1 The Benchmark Equilibrium

Here, we assume that all voter groups have identical ideological density (say, some constant $\widetilde{\phi}$) and there exist no special interest groups or lobbies. Given the latter assumptions, the electoral candidates will receive no campaign donation, such that, $C^X = C^Y = 0$ and further $\phi_j = \widetilde{\phi}$. Therefore, equation (9) will imply that,

$$p_X = \frac{1}{2} + \frac{\psi \sum_j \eta_j . \widetilde{\phi}[u_j(t_X) - u_j(t_Y)]}{\widetilde{\phi}}. \tag{11}$$

Next, using equation (4),

$$p_X = \frac{1}{2} + \frac{\psi\widetilde{\phi} \sum_j \eta_j [(1 - t_X) - \frac{1}{2}\left(t_X n - \theta_j - \frac{1}{n}\right)^2 - (1 - t_Y) + \frac{1}{2}\left(t_Y n - \theta_j - \frac{1}{n}\right)^2]}{\widetilde{\phi}}. \tag{12}$$

By utilizing equation (2), it can be deduced that, in the absence of interest groups and donations, candidate $X$ will choose $t_X$ to maximize the following expected pay-off function:

$$\text{Max}_{t_X} \ [p_X R + (1 - p_X)Q],$$

where, $p_X$ is given by equation (12). Differentiating this expression with respect to $t_X$, results in the following first-order condition:

$$\frac{(R - Q)\psi\widetilde{\phi} \sum_j \eta_j [-1 - n\left(t_X n - \theta_j - \frac{1}{n}\right)]}{\widetilde{\phi}} = 0.$$

Solving the above yields,

$$t_X^* = \frac{1}{n} . \sum_j \eta_j \theta_j. \tag{13}$$

Symmetric expressions for equilibrium tax platform can also be derived for candidate $Y$ by utilizing her probability of winning ($p_Y$). Therefore, we can write,

$$t_Y^* = \frac{1}{n} . \sum_j \eta_j \theta_j = t_X^*. \tag{14}$$

The solutions in equations (13) and (14) suggest that the policy platforms of the two electoral candidates $X$ and $Y$ converge to each other in equilibrium. Furthermore, these policy announcements turn out to be a weighted average of the policy bliss points of each voter group ($\theta_j$); the weights being the proportion of population in each group ($\eta_j$). This means that, given the preferences of each voter group in the economy, the tax platform will sway towards the group that has a larger number of people and, hence, a greater number of voters. That is, in order to win the election, each candidate being opportunistic in nature, tries to tweak her respective policy platform in favour of the voter group with a greater share of population, consequently resulting in full policy convergence or the median voter outcome.

Moreover, since the government's budget is balanced, we can write the optimal choice of the public good, $g$, of the two electoral candidates under the benchmark equilibrium as follows:

$$nt_X^* = g_X^* = \sum_j \eta_j \theta_j = nt_Y^* = g_Y^*.$$

It is evident from the above expressions that the equilibrium policy platforms chosen in the benchmark case represent a social optimum equilibrium because of the absence of interest groups and homogeneous ideological density across voter groups. The reason is that when $\phi_j = \widetilde{\phi}$, that is, when the number of swing voters is the same across all voter groups, all groups get equal weight in an electoral candidate's decision, which in effect results in maximization of the average or representative voter's utility, thus coinciding with the socially desirable outcome.

### 3.2 The Swing Voter Equilibrium

Generally, groups differ in how easily their votes can be swayed or manipulated, and therefore, office-seeking political candidates do not ascribe them equal weights in their objective function, unlike the previous case. This leads us to an alternate scenario where there exist swing voters and $\phi_j$ is different across the voter groups.

When no interest groups exist, and the ideological density across the voter groups is not uniform, using equation (9), we can write:

$$p_X = \frac{1}{2} + \frac{\psi \sum_j \eta_j.\phi_j[(1-t_X) - \frac{1}{2}\left(t_X n - \theta_j - \frac{1}{n}\right)^2 - (1-t_Y) + \frac{1}{2}\left(t_Y n - \theta_j - \frac{1}{n}\right)^2]}{\phi}. \tag{15}$$

Again, candidate $X$ will maximize the following objective function:

$$\text{Max}_{t_X}\ [p_X R + (1-p_X)Q],$$

where, $p_X$ is given by equation (15). Maximization of the above with respect to $t_X$ results in the following first-order condition:

$$\frac{(R-Q)\psi \sum_j \eta_j.\phi_j[-1 - n\left(t_X n - \theta_j - \frac{1}{n}\right)]}{\phi} = 0,$$

whose solution yields,

$$t_X^s = \frac{1}{n}.\sum_j \eta_j \phi_j \theta_j. \tag{16}$$

A symmetric equilibrium expression for candidate $Y$'s tax platform can also be derived, such that,

$$t_Y^s = \frac{1}{n}.\sum_j \eta_j \phi_j \theta_j = t_X^s. \tag{17}$$

Further, by utilizing the assumption of balanced budget of the government and rearranging equation (16) results in the following optimal choice of the public good, $g$, under the swing-voter equilibrium:

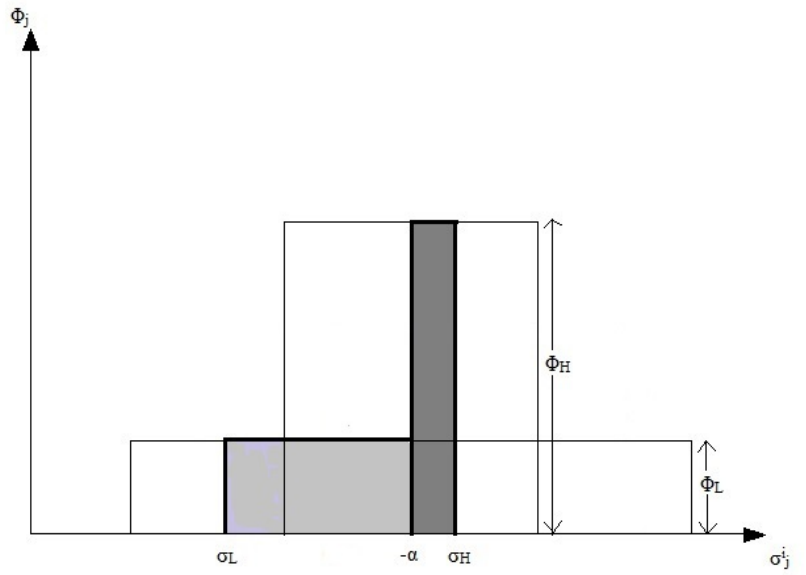
$$nt_X^s = g_X^s = \sum_j \eta_j \phi_j \theta_j = nt_Y^s = g_Y^s. \tag{18}$$

Equations (16), (17) and (18) denote the swing voter equilibrium. Since, the two candidates have symmetric objective functions, the maximization of candidate $Y$'s probability of winning also results in an identical tax being chosen by $Y$. It can be noted that the choice of tax depends on the weighted average of the policy preference parameter ($\theta_j$), with the weights now being a product of $\eta_j$ and $\phi_j$, that is, the proportion of voting population (similar to the benchmark case) and the ideological density in each group type $j$, respectively. The ideological densities, here, symbolize how responsive voters are in each group to changes in policy platforms, and in turn, how they reward it with votes during elections. This brings us to the following important result which will be used later in the analysis.

<u>Result R1</u>*: If $\phi_j$ is high, then voter group $j$ is considered to be ideologically more homogeneous, that is, it has a larger number of swing voters that makes the group more attractive for the electoral candidates. In contrast, if $\phi_j$ is low, the group is considered to be ideologically more heterogeneous, that is, there are lower number of swing voters in that group.*

The explanation for this result can be illustrated with the help of Figure 1 that depicts the distribution of $\sigma_j^i$ in the two voter groups $L$ and $H$. Both the distributions are symmetric around a zero mean value. The height of the distribution denotes the density $\phi_j$ of a voter group and quantifies the number of voters gained or lost in that group on account of a slight rise or fall in that group's economic welfare. If $\phi_j$ is high, the voter group is believed to be ideologically more homogeneous and accordingly, has a larger number of swing voters. This makes the group more appealing for office-seeking political candidates, who by tilting their policy platform in favour of this voter group, manage to gain a larger number of voters in their favour. In Figure 1, group $H$ has a higher density than group $L$. Using Lemma 1, we can say that, when both political candidates announce the same policy position, and there exist no interest groups, then the equilibrium swing voter in each group is the individual with parameter $\sigma_j = -\alpha$. Voters with $\sigma_j^i$ to the left of $-\alpha$

**Figure 1:** Change in Swing Vote Share Across Voter Groups Due to a Deviation in Policy Platform by an Electoral Candidate




Source: Own Estimation

vote for candidate $X$ while the others vote for candidate $Y$. Both the electoral candidates try to maximize their expected share of votes and, therefore, appeal to the swing voters in each group. Now, consider the case that a political candidate $X$ deviates her equilibrium policy point in favour of a higher level of public good provision financed by the imposition of larger taxes. A higher tax benefits group $H$ since group $H$ prefers a greater amount of public good provision relatively, but hurts group $L$ since group $L$ prefers lower level of public good provision.[4] Thus, as a result of this unilateral deviation, candidate $X$ gains votes from group $H$ and loses votes from group $L$. Figure 1 depicts the new swing voter in group $H$ as individual $\sigma_H$, while the new swing voter in the other group corresponds to the point $\sigma_L$. It should also be noted that the horizontal distance between $\sigma_j$ and $\alpha$ in each group is proportional to the gain or loss in utility due to this deviation. In the

[4]In the real world, this scenario can be justified by considering two distinct groups of voters existing in an economy: one, an economically weaker group of people dependent upon agriculture for their livelihoods, and second, a financially affluent group of people having opulent lifestyles. Due to such an antithetical nature of the above groups, the individuals in both these groups will categorically have divergent preferences for a public good provision. For instance, the economically weaker group would be better-off if a new irrigation facility was to be provided, since that would entail considerable utility to them. In contrast, the economically moneyed group would be more in favour of the provision of dynamic infrastructural facilities such as, a modern expressway or a brand-new airport complex coupled with ultra-modern workings and would not benefit much from an irrigation facility. Therefore, in this case, the imposition of the same level of tax on both voter groups to finance the provision of any one such public good would entail different levels of utility or disutility to the individuals.

Figure, the equilibrium is illustrated to be closest to $H$ group's bliss point and, therefore, the horizontal shift of the swing voter is quite small in that group. The electoral candidate does not have any incentive to deviate from the equilibrium when the shaded area to the right (that is, the gain in votes) is equal to the shaded area to the left (that is, the loss in votes). This implies that, in this case, the equilibrium policy must be biased in favor of group $H$. Since the proportion of swing voters in this group is greater (see the greater height of $H$ group's ideological distribution), it implies an equilibrium policy position relatively nearer to its bliss point. The opposite will be true for group $L$.

In particular, we can also analyze the impact on swing equilibrium tax platforms of electoral candidates when $\phi_L$ and $\phi_H$ tend to extreme values of 0 or 1. When $\phi_L$ tends to 0, this means that all voters in group $L$ are different in terms of their ideology, that is, are ideologically heterogeneous, implying that there are no swing voters in group $L$. Thus, the tax platform of an electoral candidate would be strongly dependent on the number of swing voters in group $H$. This is because, for an office-seeking political candidate, the number of swing voters are significant in winning the elections. Therefore, in this case, the candidate would assign a weight of zero to the utility or policy preferences of group $L$ while deciding on the equilibrium tax platform. In a similar manner, we can arrive at the intuition for the case when $\phi_H$ tends to 0, wherein the candidate will now assign a weight of zero to group $H$ and her tax policy platform would be more inclined towards the economic preferences of voter group $L$. In contrast, when $\phi_L$ tends to 1, the importance of group $L$ increases in a political candidate's decision of choosing a tax platform. Intuitively speaking, as $\phi_L$ tends to 1, group $L$ becomes ideologically more homogeneous, that is, the number of swing voters in $L$ increase. Consequently, the importance of group $L$ also rises in comparison to other voters in the economy and, hence, the electoral candidate assigns a relatively higher weight to group $L$. A similar line of reasoning can be used to obtain the results for the case when $\phi_H$ tends to 1. In addition to this, when $\phi_L$ and $\phi_H$ both tend to 1, then both the groups become equally important for the electoral candidate because the number of swing voters in each group rises and the candidate's chance of winning the election rises along with it. However, in this case the candidate has to identify the group in which the number of swing voters is relatively greater, which in turn, will be dependent on the proportion of voters in that group ($\eta_L$ and $\eta_H$). If say, $H$ type voters are greater in number, the tax platform would be biased towards them, while if $L$ type voters are more in number the candidate's tax platform will be biased towards the public good preferences of group $L$. All these results will hold for both the electoral candidates, $X$ and $Y$. The above arguments help to explain a number of our results later in the analysis.

Now, comparing the swing voter equilibrium with the benchmark equilibrium leads to the first key finding of our analysis:

**Proposition 1:** *The swing voter equilibrium tax platform (or the equilibrium level of public good*

*provision) of an office-seeking electoral candidate is always lower than or equal to the equilibrium tax platform (or the equilibrium level of public good provision) of the same candidate in case when the ideological density across voter groups is uniform and no interest groups exist (benchmark case). That is,* $t_K^s \leq t_K^*$*, where,* $K = X, Y$ *denotes the two candidates competing for electoral office in our model.*

Given $\eta_j > 0$, $\theta_j > 0$ and $\phi_j \in [0, 1]$, a comparison of equations (13) and (16) leads to the following mathematical expression:

$$\sum_j \eta_j \phi_j \theta_j \leq \sum_j \eta_j \theta_j.$$

Expanding the above, we get that,

$$\eta_L \phi_L \theta_L + \eta_H \phi_H \theta_H \leq \eta_L \theta_L + \eta_H \theta_H.$$

Rearranging, we get,

$$\frac{(\phi_L - 1)}{(1 - \phi_H)} \leq \frac{\eta_H \theta_H}{\eta_L \theta_L}.$$

Given that $\theta_H > \theta_L > 0$ and that $\phi_j$ $(j = L, H)$ is a parameter of ideological density, therefore, the left-hand side of the above inequality should always be less than the right-hand side. Utilizing this, we can postulate the above inequality as:

$$\frac{(\phi_L - 1)}{(1 - \phi_H)} \leq 0 \leq \frac{\eta_H \theta_H}{\eta_L \theta_L}.$$

This can be explained intuitively by utilizing the result $R1$. As discussed earlier, in the benchmark case the ideological density across groups is uniform (that is, $\phi_L = \phi_H = \widetilde{\phi}$) and, hence, the electoral candidate assigns equal weight (a value of 1) to each voter group in this scenario. However, in the swing voter case, voters are not identical. In fact, they have their own ideological and economic policy preferences and, hence, considering each voter group to be distinct, political candidates assign different weights to them as characterized by both the ideological density parameter and the proportion of individuals in each group. In this scenario, after an electoral candidate makes a small unilateral deviation in her equilibrium policy platform, it results in a gain and a loss in the candidate's existing vote share. The marginal benefit of policy deviation or the gain of voting support is derived from the group having a bliss point closer to the one in whose direction the policy was tilted. The marginal cost of policy deviation or the loss in vote share accrues from the loss of support from the voter group whose bliss point lies farther from the chosen policy point. In order to ensure that this gain in vote share (or, the marginal benefit) exceeds the loss (or, the marginal cost), the policy deviation has to be in favour of the economic preferences of the voter group having a higher ideological density, or a larger number of swing voters. An electoral candidate will continue to adjust her policy platform as long as the gain in vote share (marginal benefit) outweighs the loss in vote share (marginal cost). In sum, it can be stated that

the (economic) policy bliss points of both the voter groups ($\theta_j$) are compromised in the presence of swing voters in the economy, thus, reducing the level of tax platform and corresponding level of public good provision in comparison to the benchmark scenario. Put differently, the opportunism of the political candidates becomes more prevalent in the swing voter equilibrium relative to the benchmark case since, greater the proportion of swing voters in a group, greater the possibility of manipulating their votes in the political candidate's favour.

Note that, the result in Proposition 1 holds only when the ideological density is different across the groups, that is, when $\phi_H \neq \phi_L$. In contrast, when $\phi_H = \phi_L$, the tax in the swing voter equilibrium collapses to the benchmark case because in this case, all the (swing) voters are treated as identical by the electoral candidate, irrespective of which voter group they belong to. Similar results hold for the political candidate $Y$.

We now introduce the role of interest groups in our model who can lobby with the political candidates to tilt policy platforms in their favour by offering campaign donations or contributions to them. Campaign contributions help the political candidate in financing their electoral campaigns, which in turn, raises the candidate's average popularity across the voting population. This helps in increasing the electoral candidate's probability of win.

### 3.3 Equilibrium Solution for Campaign Contributions

We now derive the solutions for the lobbying stage, where there are two groups of voters $j = L, H$ who may organize and form lobbies in order to influence the electoral candidates. As explained before, $O_j \in [0, 1]$ is the fraction which represents the organizational strength of the interest group so formed, where $O_j = 1$ describes complete organization within a group of voters, that is, no free-riding amongst the voters in that group, whereas $O_j = 0$ describes no organization among voters within a group, that is, a situation where everyone is free-riding. Additionally, $0 < O_j < 1$ depicts a situation where a fraction of voters in a group $j$ are organized, whereas, the remaining voters in that group free-ride. If a group $j$ gets organized into a lobby, then a member of this lobby makes a contribution of $C_j^P$ which is a payment to the electoral candidate $P$ ($P = X, Y$) and $C_j^P > 0$. These contributions can be comprehended both in cash and in kind. Thus, the total contributions to a party or a candidate $P$ can be written as:

$$C^P = \sum_j O_j \eta_j C_j^P. \tag{19}$$

In order to calculate an individual contribution from group $j$ to party $P$ ($C_j^P$), the interest group's maximization problem has to be considered. An organized group's utility depends on the equilibrium policy besides the amount of political contributions provided to electoral candidates. Thus, it is assumed to take the following form:

$$p_X u_j(t_X) + (1 - p_X) u_j(t_Y) - \frac{1}{2}[(C_j^X)^2 + (C_j^Y)^2]. \tag{20}$$

The quadratic form of the cost function captures that contributions not only involve a monetary transfer, but also the personal involvement of organized voters, thus implying convexity of the cost of contributing. Therefore, an organized group $j$ will maximize its expected utility, which is given by:

$$\text{Max}_{C_j^X, C_j^Y} \; E(v_j) = p_X u_j(t_X) + (1 - p_X) u_j(t_Y) - \frac{1}{2}[(C_j^X)^2 + (C_j^Y)^2], \tag{21}$$
$$\text{subject to } C_j^X, C_j^Y > 0,$$

where, $p_X$ is given by equation (9). Differentiating $E(v_j)$ with respect to $C_j^X$, we get the following Kuhn-Tucker conditions:

$$\frac{\partial p_X}{\partial C_j^X}[u_j(t_X) - u_j(t_Y)] - C_j^X \leq 0, \tag{22}$$

and,

$$\left[\frac{\partial p_X}{\partial C_j^X}[u_j(t_X) - u_j(t_Y)] - C_j^X\right] C_j^X = 0. \tag{23}$$

Using equations (9) and (19), this can be expressed as:

$$\frac{\partial p_X}{\partial C_j^X} = \frac{h\psi\delta\beta_X O_j \eta_j}{\phi}.$$

Therefore, using the above, the solution for individual contributions from group $j$ to candidate $X$ will be:[5]

$$C_j^X = \text{Max}\left\{0, \frac{h\psi\delta\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi}\right\}. \tag{24}$$

In a similar manner, the solution for individual contributions from group $j$ to candidate $Y$ can be derived to be:

$$C_j^Y = \text{Max}\left\{0, \frac{h\psi\delta\beta_Y O_j \eta_j [u_j(t_Y) - u_j(t_X)]}{\phi}\right\}. \tag{25}$$

From the above, it can be deduced that a lobby group donates only to the political candidate whose policy platform yields to the group the highest utility and never contributes to more than one political candidate.[6]

[5]The solution $C_j^X = 0$ reduces equation (22) to $\frac{\partial p_X}{\partial C_j^X}[u_j(t_X) - u_j(t_Y)] \leq 0$. And since, $\frac{\partial p_X}{\partial C_j^X} > 0$, we can conclude that, $[u_j(t_X) - u_j(t_Y)] \leq 0$. This means that, if an individual member of the lobby group supporting electoral candidate $X$ derives lesser (or equal) utility from $X$'s policy choice as compared to $Y$'s policy choice, then he/she finds no incentive to continue supporting candidate $X$. As a result, the individual contributions to $X$ in this scenario tend to zero. In sum, a lobby member will make positive contributions only when he/she derives a higher utility from the policy platform of the electoral candidate that their respective lobby group is supporting. Similar reasoning holds when $C_j^Y = 0$.

[6]The result that a lobby group provide contributions to only one political party is fairly conventional. It has been pointed out and discussed in Austen-Smith (1987), Baron (1988), Persson and Tabellini (2002), Hall and Deardorff (2006) and Le and Yalcin (2018) despite using different modelling frameworks. According to Baron (1988), there exists empirical evidence depicting that most PACs (Political Action Committees) contribute to only one political candidate.

### 3.4 Policy Equilibrium under Electoral Competition in the Presence of Both Interest Groups and Swing Voters

In this sub-section, we solve for the equilibrium of the third specification of the model, that is, the policy platform of the political candidates under electoral competition in the presence of both the interest groups and swing voters. For this, we focus on the solution for the first stage of the game. In this stage of the game, each electoral candidate maximizes her expected payoffs she receives from engaging in electoral competition. Using equation (2), the objective function of candidate $X$ can be written as follows:

$$\text{Max}_{t_X}\ p_X[(1-\beta_X)C^X + R] + (1-p_X)[(1-\beta_X)C^X + Q].$$

The above can also be expressed as:

$$\text{Max}_{t_X}\ p_X(R-Q) + (1-\beta_X)C^X + Q, \quad \text{subject to} \quad 0 \leq t_X \leq y.$$

As derived in the previous sub-section, any individual member of a lobby will donate to a candidate whose choice of policy platform gives her a higher utility or payoff. Here, we assume that both the interest groups ($j = L, H$) attain greater utility with $X$'s policy platform as compared to $Y$'s policy platform and, hence, donate only to $X$ and not to $Y$.[7] Therefore, using equation (19) and (24), the aggregate contributions to $X$ can be written as:

$$C^X = \frac{h\psi\delta\beta_X \sum_j O_j^2\eta_j^2[u_j(t_X) - u_j(t_Y)]}{\phi}.$$

As explained in the previous section, the total contributions received by a political candidate $X$ will be positive only if $u_j(t_X) > u_j(t_Y)$, that is, a voter $j$ would contribute only when she receives a greater utility from candidate $X$'s policy choice in comparison to candidate $Y$'s policy choice. Furthermore, the above expression indicates that candidate $X$ garners more contributions if she indulges in lesser embezzlement of funds and if political campaign expenditure is more effective at swaying votes in her favour. Given that $C^Y = 0$, using the above expression yields the following:

$$(\beta_X C^X - \beta_Y C^Y) = \frac{h\psi\delta\beta_X^2 \sum_j O_j^2\eta_j^2[u_j(t_X) - u_j(t_Y)]}{\phi}.$$

[7] In our model, we have assumed that lobby groups have an influence motive for donating to a political candidate and not an electoral motive and, therefore, both lobby groups can donate to the same political candidate bearing in mind their respective policy preferences. The receipt of monetary donations from two antithetical lobby groups by one political candidate strengthens our analysis further by adding another conflicting force besides the familiar centripetal and centrifugal forces, that help shape the final equilibrium policy choices of the electoral candidates.

Utilizing equation (9) and substituting the above expression into it, the objective function for candidate $X$ can be written as:

$$\text{Max}_{t_X}\ \Big[\frac{1}{2}+\frac{\psi}{\phi}[u(t_X)-u(t_Y)+\frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2(u_j(t_X)-u_j(t_Y))]\Big].(R-Q)$$
$$+(1-\beta_X)\frac{\psi}{\phi}h\delta\beta_X\sum_j O_j^2\eta_j^2[u_j(t_X)-u_j(t_Y)]+Q.$$

Next, using equation (4) and differentiating the above with respect to $t_X$ yields the first-order condition as:

$$[-nt_X\phi+\sum_j\eta_j\phi_j\theta_j+\frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2(\theta_j-nt_X)].(R-Q)+(1-\beta_X)\frac{\psi}{\phi}h\delta\beta_X\sum_j O_j^2\eta_j^2(\theta_j-nt_X)=0.$$

Rearranging the above and using equation (16) results in the following expression for candidate $X$'s optimal tax platform under electoral competition in the presence of swing voters and lobbying activities when both lobby groups donate only to $X$:

$$t_X^e=\frac{(R-Q)\left[t_X^s+\frac{\psi}{\phi}h^2\delta^2\beta_X^2\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]+\left[(1-\beta_X)\beta_X h\delta\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{(R-Q)\left[\phi+\frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right]+[(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]},\tag{26}$$

where, $t_X^s$ is the swing voter equilibrium tax platform of candidate $X$ (see equation (16)). The above expression denotes that the equilibrium tax platform of an electoral candidate $X$ who receives campaign contributions from two different lobby groups is found to be dependent upon the swing policy outcome as well as the influence exerted by the two opposing lobby groups having different policy bliss points via their respective campaign donations.

In addition, when both lobby groups donate only to $X$, candidate $Y$'s maximization problem becomes identical to that of the swing voter's case and, hence, the equilibrium tax platform of candidate $Y$ coincides with the swing voter equilibrium presented in equation (17), that is,

$$t_Y^e=t_Y^s=\frac{1}{n}.\sum_j\eta_j\phi_j\theta_j.$$

In the same vein, when both lobby groups donate to $Y$, the maximization of candidate $Y$'s objective function results in the following expression for the choice of optimal tax platform of candidate $Y$ under electoral competition in the presence of swing voters and lobbying activities:

$$t_Y^e=\frac{(R-Q)\left[t_Y^s+\frac{\psi}{\phi}h^2\delta^2\beta_Y^2\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]+\left[(1-\beta_Y)\beta_Y h\delta\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{(R-Q)\left[\phi+\frac{\psi}{\phi}h^2\delta^2\beta_Y^2\sum_j O_j^2\eta_j^2\right]+[(1-\beta_Y)\beta_Y h\delta\sum_j O_j^2\eta_j^2]},\tag{27}$$

where, $t_Y^s$ is the swing voter equilibrium tax platform of candidate $Y$ (see equation (17)). The above expression denotes that the equilibrium tax platform of an electoral candidate $Y$ who receives campaign contributions from two different lobby groups is found to be dependent upon the swing

policy outcome as well as the influence exerted by the two opposing lobby groups having different policy bliss points via their respective campaign donations.

Further, the maximization problem of candidate $X$ is reduced to the one seen in the swing voter equilibrium and leads to candidate $X$ choosing the swing voter tax as the equilibrium policy even in the presence of interest groups. That is, candidate $X$'s equilibrium tax in this case can be written as follows:

$$t_X^e = t_X^s = \frac{1}{n}.\sum_j \eta_j \phi_j \theta_j.$$

As is clear from the above calculations, the tax platforms of the two political candidates do not converge to each other when two opposing lobby groups provide campaign contributions to only one electoral candidate. Moreover, it can be noted that the equilibrium tax platform of the electoral candidate receiving the donations does not converge to the mean voter as well. Instead, it is now influenced by various parameters directly or indirectly related to campaign donations and political expenditure of the candidate, such as the honesty parameter, effectiveness of campaign expenditure, popularity parameter, proportion of unorganized voters, organizational strength of lobby groups and difference between win and loss payoff of the candidate from contesting elections. The explicit impact of a change in all these parameters on the equilibrium tax platforms is discussed in detail in the sub-section on comparative statics (see sub-section 3.5.2).

In what follows immediately, we compare the tax platform under electoral competition with interest groups and swing voters ($t_X^e$) to the one under the benchmark case ($t_X^*$) and also with the one under the swing voter case ($t_X^s$).

# 4 A Comparison of Policy Platforms Across Different Equilibria

In this section, we first compare the tax platform of an electoral candidate in the benchmark case to the tax platform under electoral competition with interest groups and swing voters. Using equations (13) and (26), the difference in the equilibrium tax levels can be expressed as:

$$t_X^e - t_X^* = \frac{(R-Q)\left[t_X^s + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] + \left[(1-\beta_X)\beta_X h\delta\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]} - t_X^*,$$

which implies that,

$$t_X^e - t_X^* = \frac{(R-Q)[t_X^s - \phi t_X^*] + \left[(R-Q)\frac{\psi}{\phi}h\delta\beta_X + (1-\beta_X)\right](\beta_X h\delta)\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^*\sum_j O_j^2\eta_j^2\right]}{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]}.$$

Given that $R > Q$ and $0 \leq \beta_X \leq 1$, it can be inferred from the above that the sign of left-hand side depends upon the sign of two terms in the right-hand side, that is, $[t_X^s - \phi t_X^*]$ and $\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^*\sum_j O_j^2\eta_j^2\right]$.

Let us consider the term $[t_X^s - \phi t_X^*]$. Using equation (13), this can be written more elaborately as:

$$\left[\frac{(\eta_L\phi_L\theta_L}{n} + \frac{\eta_H\phi_H\theta_H)}{n} - \phi\frac{(\eta_L\theta_L + \eta_H\theta_H)}{n}\right] = \frac{[\eta_L\theta_L(\phi_L - \phi) + \eta_H\theta_H(\phi_H - \phi)]}{n}.$$

Using the definition of average ideology $\phi$ and substituting in the above, we get,

$$\frac{[\eta_L\theta_L(\phi_L - \eta_L\phi_L - \eta_H\phi_H) + \eta_H\theta_H(\phi_H - \eta_L\phi_L - \eta_H\phi_H)]}{n},$$

which, when rearranged, yields,

$$\frac{[\eta_L\theta_L(\phi_L(1-\eta_L) - \eta_H\phi_H) + \eta_H\theta_H(\phi_H(1-\eta_H) - \eta_L\phi_L)]}{n}.$$

Since $\eta_L + \eta_H = 1$, we can write the above expression as:

$$\frac{[\eta_L\theta_L(\eta_H\phi_L - \eta_H\phi_H) + \eta_H\theta_H(\eta_L\phi_H - \eta_L\phi_L)]}{n},$$

which can be further simplified to get,

$$\frac{[\eta_H\eta_L(\phi_H - \phi_L)(\theta_H - \theta_L)]}{n} \lessgtr 0 \text{ if } \phi_H \lessgtr \phi_L, \tag{28}$$

given that, $\theta_H > \theta_L$, that is, voter group $H$ prefers a higher level of public good provision relative to voter group $L$. In general, this term represents the impact of voter's ideological heterogeneity on the benchmark equilibrium tax and the equilibrium tax in the presence of swing voters and organized interest groups. Categorically speaking, it can be inferred from this expression that a higher density of swing population in a voter group shifts the swing voter tax platform in favour of that group's public good preferences as compared to the equilibrium tax platform in the benchmark case. Thus, this expression helps in segregating the workings of the swing voter effect on the equilibrium policy choices.

Now consider the second term, $\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^* \sum_j O_j^2\eta_j^2\right]$. This term accounts for the influence of distinct lobby groups on the equilibrium tax choice in the presence of swing voters and lobby groups and in the benchmark case. Expanding the summation across voter groups $L$ and $H$, we have:

$$\frac{O_L^2\eta_L^2\theta_L}{n} + \frac{O_H^2\eta_H^2\theta_H}{n} - t_X^*(O_L^2\eta_L^2 + O_H^2\eta_H^2).$$

Again using equation (13), we get that:

$$O_L^2\eta_L^2\left[\frac{\theta_L}{n} - \frac{\eta_L\theta_L}{n} - \frac{\eta_H\theta_H}{n}\right] + O_H^2\eta_H^2\left[\frac{\theta_H}{n} - \frac{\eta_L\theta_L}{n} - \frac{\eta_H\theta_H}{n}\right].$$

This can be further simplified to the following expression:

$$O_L^2\eta_L^2\left[\frac{\theta_L}{n}(1-\eta_L) - \frac{\eta_H\theta_H}{n}\right] + O_H^2\eta_H^2\left[\frac{\theta_H}{n}(1-\eta_H) - \frac{\eta_L\theta_L}{n}\right].$$

Since only two groups $L$ and $H$ exist in the economy, therefore, we can write $(1-\eta_H)=\eta_L$ and $(1-\eta_L)=\eta_H$. Substituting this, and simplifying, we get:

$$\frac{\eta_L\eta_H}{n}[\theta_H-\theta_L][O_H^2\eta_H-O_L^2\eta_L]\lesseqgtr 0 \text{ if } O_H^2\eta_H\lesseqgtr O_L^2\eta_L, \tag{29}$$

given that, $\theta_H>\theta_L$. This suggests that the equilibrium tax platforms react to the relative organizational effect of lobby groups $H$ and $L$ such that, the dominant lobby group takes precedence over the weakly organized lobby group while manipulating the electoral candidate's equilibrium policy decisions in their favour. Thus, this helps to segregate the impact of lobby effect on the electoral candidate's equilibrium policy choices.

We can now infer the sign of $t_X^e-t_X^*$ from the expressions (28) and (29) to get that:

**Proposition 2:** *The equilibrium tax chosen by candidate $X$ under electoral competition in the presence of swing voters and interest groups ($t_X^e$) will be more than the equilibrium tax in the benchmark case ($t_X^*$) if the following two sufficiency conditions hold: $\phi_H>\phi_L$ and $O_H^2\eta_H>O_L^2\eta_L$. On the other hand, if $\phi_H<\phi_L$ and $O_H^2\eta_H<O_L^2\eta_L$, $t_X^e<t_X^*$. Similar results hold for candidate $Y$ as well.*

When $\phi_H>\phi_L$ and $O_H^2\eta_H>O_L^2\eta_L$, $t_X^e>t_X^*$, it implies that the amount of public good provision (and, hence, the tax policy) will be lower in the benchmark case as compared to a scenario where both swing voters and interest groups exist. In this case, voter group $H$ becomes more important to the political candidate, both as a swing voter group as well as a lobby group. This can be seen from the first condition (in equation (28)) which entails a lower ideological density of group $L$ relative to group $H$, in turn, implying a larger number of swing voters in group $H$. Using result $R1$, it can be inferred that the dominant swing voter effect in group $H$ will make the candidate tilt towards the economic preferences of voter group $H$, which involves a higher level of public good provision and, hence, a higher tax platform. Moreover, the second condition (in equation (29)) clearly shows that when lobby group $H$ is stronger relative to lobby group $L$, whether it be in terms of proportion of people ($\eta$) or in terms of organizational strength ($O_j$), this further induces the electoral candidate to choose a tax in favour of group $H$'s policy bliss point. Given that the interest groups are a source of campaign money, a stronger lobby group, therefore, develops into a source of secure funding for the political candidate in question. When both these effects work in the same direction, the equilibrium tax platform chosen by a political candidate in the presence of swing voters and lobby groups exceeds the equilibrium tax platform chosen by the same candidate in the absence of these two effects. In contrast, when $\phi_H<\phi_L$ and $O_H^2\eta_H<O_L^2\eta_L$, the opposite result holds. That is if, besides a strongly organized interest group $L$, when there also exist a larger number of swing voters in group $L$, it will result in a reduction in the equilibrium tax platform (and, hence, the level of public good provision) relative to the benchmark case; which

will be in alignment with the public good preferences of group $L$. Thus, in these cases the both the swing voter effect and the relative organizational effect of lobbies effect work in the same direction. However, when $\phi_H > \phi_L$ ($\phi_H < \phi_L$) and $O_H^2\eta_H < O_L^2\eta_L$ ($O_H^2\eta_H > O_L^2\eta_L$), the results are ambiguous and depend upon whether the swing voter effect or the relative organizational effect of lobbies dominates.

We next compare the equilibrium tax platform announced by electoral candidate $X$ in the swing voter case (equation (16)) and the case in which both swing voters and interest groups are present (see equation (26)). To compare $t_X^e$ with $t_X^s$, we use equations (16) and (26) and derive that,

$$(t_X^e - t_X^s) = \frac{(R-Q)\left[t_X^s + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] + \left[(1-\beta_X)\beta_X h\delta\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]} - t_X^s,$$

where, the right-hand side can be expressed as:

$$\frac{(R-Q)\left[t_X^s + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] + \left[(1-\beta_X)\beta_X h\delta\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] - (R-Q)\phi.t_X^s}{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]}$$
$$-\frac{(R-Q).t_X^s\left[\frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2].t_X^s}{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]}.$$

Since, $(R-Q) > 0$ and $0 \leq \beta_X \leq 1$, the denominator is positive and, therefore, we only focus on the terms in the numerator, which when rearranged yield the following:

$$(R-Q)t_X^s[1-\phi] + (R-Q)\frac{\psi}{\phi}h^2\delta^2\beta_X^2\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s.\sum_j O_j^2\eta_j^2\right] + (1-\beta_X)\beta_X h\delta\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s.\sum_j O_j^2\eta_j^2\right],$$

where $\phi$ is the average ideology of the population as discussed earlier. In this case, a sufficient condition for $(t_X^e - t_X^s) > 0$ is $(1-\phi) > 0$ and $\left[\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s.\sum_j O_j^2\eta_j^2\right] > 0$.

First, since $\phi = \sum_j \eta_j\phi_j$ ($j = L, H$) is the average ideology of the population, and $\eta_j \in [0,1]$ and $\phi_j \in [0,1]$, therefore $(1-\phi)$ will always be non-negative. Next, we expand the second term as follows:

$$\frac{O_L^2\eta_L^2\theta_L}{n} + \frac{O_H^2\eta_H^2\theta_H}{n} - t_X^s(O_L^2\eta_L^2 + O_H^2\eta_H^2) = O_L^2\eta_L^2\left[\frac{\theta_L}{n} - t_X^s\right] + O_H^2\eta_H^2\left[\frac{\theta_H}{n} - t_X^s\right]. \tag{30}$$

In the above expression, $\frac{\theta_H}{n}$ and $\frac{\theta_L}{n}$ are nothing but first-best solutions for tax policy for the $H$ and $L$ voter groups respectively (see equation (5)). A swing voter equilibrium provides a solution for tax policy platform such that electoral candidates only take into account the proportion of swing voters across groups. However, in comparison, the scenario involving both swing voters and interest groups results in the formulation of another effect at work that corresponds with the presence of two different lobby groups comprising of voters from different groups $L$ and $H$. These interest groups want their first-best tax to be implemented as a final policy choice by the winner of the election and, therefore, organize as lobbies to influence the electoral candidates. By using

equation (16), we can express the right-hand side of equation (30) as:

$$O_L^2\eta_L^2\left[\frac{\theta_L}{n}-\frac{\eta_L\phi_L\theta_L}{n}-\frac{\eta_H\phi_H\theta_H}{n}\right]+O_H^2\eta_H^2\left[\frac{\theta_H}{n}-\frac{\eta_L\phi_L\theta_L}{n}-\frac{\eta_H\phi_H\theta_H}{n}\right].$$

The first term in the first square bracket above denotes the first-best solution of tax that interest group $L$ wants to lobby for, and this multiplied by $O_L^2\eta_L^2$ represents the organizational strength of group $L$ in the economy. Following this reasoning, the second and third terms depict the reduction in organizational strength of group $L$ due to the presence of swing voters in voter groups $L$ as well as $H$. A greater proportion of swing voters in the economy reduce the effectiveness of lobby groups as a secure source of funding from the point of view of an electoral candidate, hence, creating more scope for a deviation from the first-best policy choice of that lobby group. Similarly, the first term in the second square bracket denotes the first-best solution for group $H$ which when multiplied by the measure of its organizational strength $O_H^2\eta_H^2$ depicts the strength of lobby group $H$ in the economy. Again, the magnitude of this strength is lowered due to the presence of swing voters in voter group $H$ as well as voter group $L$, thereby causing a deviation from the first-best policy equilibrium of group $H$.

Equation (30) can be further modified by utilizing the balanced budget of the government where $nt_X = g_X$ and can be written as follows:

$$O_L^2\eta_L^2\underbrace{\left[\frac{\theta_L-g_X^s}{n}\right]}_{+/-}+O_H^2\eta_H^2\underbrace{\left[\frac{\theta_H-g_X^s}{n}\right]}_{+/-}.$$

Using this expression, the relation between $t_X^s$ and $t_X^e$ can be presented more formally through the following proposition.

**Proposition 3:** *Under electoral competition, if $\theta_H > \theta_L > g_X^s$, then the equilibrium tax platform (or the amount of public good provision) chosen by an opportunistic electoral candidate ($X$ or $Y$) in the presence of both interest groups and voters with differing ideological densities (that is, $t_X^e$) will be greater than the choice of the tax platform (or the amount of public good provision) in the swing voter equilibrium (that is, $t_X^s$). Also, when $g_X^s > \theta_H > \theta_L$, then it will result in a reduction in $t_X^e$, and if $\theta_H > g_X^s > \theta_L$, the equilibrium level of tax platform and the level of public good provision will be biased towards the economic preferences of the dominating interest group.*

From the above discussion, if $\theta_H > \theta_L > g_X^s$ and $(1-\phi) > 0$, then $t_X^e > t_X^s$. It means that, if an electoral candidate's level of provision of public good (and the amount of tax) under the swing voter equilibrium is lower than what is preferred by all the voter groups ($H$ and $L$), then an introduction of distinct lobby groups (comprising of voters from groups $L$ and $H$) into the existing framework induces the candidate to provide a higher level of public good along with a higher level of tax platform. Additionally, as both $\phi_L$ and $\phi_H$ tend to zero, the term $(1-\phi)$ tends

to 1 indicating that the impact of lobby groups on the equilibrium tax platform gets enhanced.[8] Thus, in the absence of swing voters in any voter group (that is, with ideologically heterogeneous population), only the lobbying effect will prevail, and in accordance with the economic preferences of both interest groups, the electoral candidate will opt for a higher level of equilibrium tax and public good provision. Moreover, when both $\phi_L$ and $\phi_H$ tend to 1, the term $(1-\phi)$ tends to 0, in turn, implying that the effect of lobbying on equilibrium choice of tax platform still emerges as the dominant force (that is, $t_X^e > t_X^s$), though it does get slightly moderated on account of the presence of swing voters in the economy unlike the previous case.[9] Alternatively, if $g_X^s > \theta_H > \theta_L$, then there will be a downward pressure on the level of public good provision (and, hence, the equilibrium tax platform) of the electoral candidate in the presence of interest groups relative to the swing voter case. This is because, if the level of public good provision in the swing voter equilibrium exceeds what the voter groups $L$ and $H$ in the economy prefer, the introduction of interest groups (comprising of voters from groups $L$ and $H$) into the framework creates a lobbying effect that supports a lower level of public good provision (and tax platform) in consonance with both lobby group's economic preferences. The degree of success of this lobbying activity will eventually depend on the voter's ideological preferences, that is, the total proportion of swing voters in the economy and the average ideology $(1-\phi)$ of the population. For instance, when $\phi_L = \phi_H = 1$, that is, when the relative swing voter effect is quite intense initially, then an introduction of lobbies creates a very acute impact on the electoral candidate and lowers the candidate's equilibrium tax platform. Thus, the impact of lobbies emerges as the strongest in this case. However, if $\phi_L \neq \phi_H \neq 1$, the relative swing voter effect is not so strong initially, and the electoral candidate does not want to compromise this limited swing vote share by giving in completely to the lobby groups' efforts. Hence, the impact on equilibrium tax platform is ambiguous in this case. Notably, if the negative lobbying effect overpowers the term $(1-\phi)$, then $t_X^e < t_X^s$.

Finally, if $\theta_H > g_X^s > \theta_L$, then the two interest groups become antithetical in nature, where, interest group $H$ will lobby to increase the public good provision while interest group $L$ will lobby to reduce the level of public good provision.[10] In this case, the result will depend on whichever lobby is stronger in terms of organizational strength. For instance, if lobby $H$ is stronger than lobby $L$, then $t_X^e$ will exceed $t_X^S$ and if lobby $L$ is stronger than lobby $H$, then $t_X^s$ will exceed $t_X^e$. This can be shown by considering a special case, wherein, each group comprises of exactly half of the total population and also has an identical ideological density value of 1, such that the economy

[8]Since $(1-\phi) = (1-\eta_L\phi_L - \eta_H\phi_H)$, therefore, $\phi_L$ and $\phi_H$ tending to zero will clearly imply that the term $(1-\phi)$ will tend to 1.

[9]Since $(1-\phi) = (1-\eta_L\phi_L - \eta_H\phi_H)$, therefore, $\phi_L$ and $\phi_H$ tending to 1 means that the above term becomes $(1-\eta_L-\eta_H)$. And because $\eta_L + \eta_H = 1$, the above expression reduces to 0.

[10]As mentioned before, a real world illustration for such a scenario would be the one with two economically divergent groups of voters who want such a public good to be provided that conforms to their respective economic preferences as well as their level of incomes.

now constitutes a completely ideologically homogeneous population. In mathematical terms we can say that in this case, $\eta_L = \eta_H = 0.5$ and $\phi_L = \phi_H = 1$, which implies that $(1 - \phi) = 0$. By using equation (16), $\left[\frac{\sum_j O_j^2 \eta_j^2 \theta_j}{n} - t_X^s . \sum_j O_j^2 \eta_j^2\right]$ can be written as follows:

$$0.25O_L^2 \left[\frac{\theta_L}{n} - 0.5(\frac{\theta_L}{n} + \frac{\theta_H}{n})\right] + 0.25O_H^2 \left[\frac{\theta_H}{n} - 0.5(\frac{\theta_L}{n} + \frac{\theta_H}{n})\right].$$

Since by assumption, $\theta_H > \theta_L$, therefore, we can say that, $\frac{\theta_H}{n} > \frac{\theta_L}{n}$. Or, $\frac{\theta_H}{n} = \frac{\theta_L}{n} + \epsilon$, where, $\epsilon > 0$. Substituting this in the above expression, we get,

$$0.25O_L^2 \left[\frac{\theta_L}{n} - 0.5\left(\frac{2\theta_L}{n}\right) - 0.5\epsilon\right] + 0.25O_H^2 \left[\frac{\theta_H}{n} - 0.5\left(\frac{2\theta_H}{n}\right) + 0.5\epsilon\right],$$

which can be simplified and expressed as:

$$0.125\epsilon \underbrace{[O_H^2 - O_L^2]}_{+/-}.$$

From this, one can note that the equilibrium level of tax platform and the public good provision in this special case depends on the organizational strength of the two lobby groups $L$ and $H$. If lobby $L$ is stronger, it manages to reduce the equilibrium tax level (and, hence, the level of public good provision) in its favour (that is, $t_X^e < t_X^s$) while if lobby $H$ is stronger, it succeeds in raising the equilibrium tax level (and, hence, the level of public good provided) in its favour (that is, $t_X^e > t_X^s$). Furthermore, the extent of deviation of tax platform in the presence of swing voters and interest groups from the swing voter case also depends on the magnitude of variation in economic policy preferences across voter groups (as denoted by $\epsilon$). This term actually captures the distinct nature of economic policy preferences of different voter groups. And, thus, greater the magnitude of this variation (whether it be negative or positive), greater will be the digression from the swing voter equilibrium tax platform.

# 5 Comparative Statics

In this section, we focus on the analysis of some comparative statics of campaign contributions and equilibrium tax platforms with respect to various parameters employed in our model. The following sub-sections provide the results of the analysis.

## 5.1 Comparative Statics for Campaign Contributions

We first derive some comparative statics results for the aggregate donations made by the interest groups to political candidates to attain a better understanding of the workings of our model via a change in the exogenous parameters of our model.

### 5.1.1 Comparative Statics with respect to Honesty Parameter, $\beta_K$ $(K = X, Y)$

**Proposition 4:** *When a member of an interest group $j \in \{L, H\}$ derives greater utility from candidate X's policy platform relative to Y's, that is, $u_j(t_X) > u_j(t_Y)$, an increase in the honesty parameter of candidate X (that is, $\beta_X$), results in higher donations by that lobby group member to candidate X. Similar result holds for candidate Y as well.*

This can be explained mathematically as follows. Since,

$$C_j^X = \text{Max}\left\{0, \frac{h\psi\delta\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi}\right\}.$$

Therefore,

$$\frac{dC_j^X}{d\beta_X} = \frac{h\psi\delta O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi} > 0. \tag{31}$$

Intuitively, a higher $\beta_X$ implies that an electoral candidate is more honest in spending the contributions received for electoral campaigning, and there is lower diversion of money for private use. This leads to the members of the interest group or lobby to increase their donations to the candidate because they provide money to an electoral candidate for the purpose of campaign spending, which in turn, is used to influence voters, and hence, win elections. If the candidate spends a larger proportion of the money on election campaigns, the marginal benefit derived by a member of the interest group would be higher due to a lower leakage of money. In conclusion, if $[u_j(t_X) - u_j(t_Y)] > 0$,[11] then greater the value of $\beta_X$, greater will be the donations to candidate $X$, since now, money will be more effectively utilized by $X$ for the purpose of campaigning for elections.[12]

### 5.1.2 Comparative Statics with respect to Campaign Spending Efficiency Parameter, $h$

**Proposition 5:** *When a member of an interest group $j \in \{L, H\}$ derives a greater utility from candidate X's policy platform relative to Y's policy platform, that is, $u_j(t_X) > u_j(t_Y)$, an increase*

[11] As explained in sub-section 3.3.3, when $u_j(t_X) < u_j(t_Y)$, an individual member of a lobby group derives a lower utility from candidate $X$'s policy choice relative to candidate $Y$'s policy choice and, therefore, finds no incentive to contribute towards candidate $X$, and as a consequence, the individual contributions to $X$ in this scenario tend to zero. It is thus quite apparent that the individual contributions received by electoral candidate $X$ ($C_j^X$) cannot be influenced by a change in any of the exogenous parameters considered in this and any of the subsequent comparative statics results.

[12] This result is in contrast with the result proposed by Le and Yalcin (2018) where they state that a greater embezzlement of campaign funds creates a less favourable position for the lobbies as this entails greater liability on the latter in terms of providing higher contributions to the political parties. The reason is the sequential game effect in which the lobbies, in order to be incentive compatible, have to adequately compensate the political party for the loss in utility it derives on account of swaying its policies in favour of the lobby's desired policy. In our model, we abstract from this effect because we assume that both the electoral representatives do not get any utility out of their own policy choices and are purely office-seeking by nature.

*in the efficiency of campaign spending (h) by a political candidate K (K = X, Y) entails higher donations being provided to that political candidate by the members of her supporting lobby group j.*

Mathematically, this can be shown as:

$$\frac{dC_j^X}{dh} = \frac{\psi\delta\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi} > 0. \tag{32}$$

The intuitive explanation for this result is fairly straightforward. When the members of an interest group perceive that the expenditure on electoral campaigning by a political candidate exerts a higher influence on unorganized voters, which in turn, results in the candidate garnering a greater vote share and winning the election, their willingness to donate to that candidate rises. This is because, given the increased significance of campaign money use during election by the political candidates, the individual members of interest groups know that now they can raise their chances of lobbying success and subsequent implementation of their preferred policy position post election.

### 5.1.3 Comparative Statics with respect to Ideological Density Parameter, $\phi_j$ ($j = L, H$)

**Proposition 6:** *When $u_j(t_X) > u_j(t_Y)$, a member of an interest group $j \in \{L, H\}$ will provide lesser contributions to candidate X in equilibrium if she belongs to the voter group comprising of a greater number of swing voters (that is, a higher $\phi_j$).*

By using $\phi = \sum_j \eta_j . \phi_j$, which is the average or mean ideology of the population and differentiating the solution for aggregate contributions by $\phi_j$, we get,

$$\frac{dC_j^X}{d\phi_j} = \frac{[0 - h\psi\delta\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]\eta_j}{\phi^2} < 0,$$

which after rearranging implies that,

$$\frac{dC_j^X}{d\phi_j} = -\frac{h\psi\delta\beta_X O_j \eta_j^2 [u_j(t_X) - u_j(t_Y)]}{\phi^2} < 0. \tag{33}$$

As explained in Result $R1$, an increase in the ideological density within a group ($\phi_j$) indicates a rise in the number of swing voters in that group. Now, the political candidates can increase their vote share by targeting swing voters via a deviation in their respective equilibrium policy platforms, thereby limiting their absolute dependence on campaign spending to woo uninformed voters. This reduction in the significance of campaign expenditure as a channel to sway voters consequently diminishes the incentive of an individual member of a lobby group to provide greater campaign contributions to the political candidates. Hence, for lobby group members, the marginal cost of donating exceeds the marginal benefit from donating when the proportion of swing voters

increases in the economy, in turn, reducing the aggregate supply of campaign donations to an electoral candidate in equilibrium.

### 5.1.4 Comparative Statics with respect to Proportion of Uninformed Voters, $\delta$

**Proposition 7:** *A rise in the proportion of uninformed (or unorganized) voters ($\delta$) in the economy leads to an increase in individual member contributions from $j$th lobby group ($j \in \{L, H\}$) to candidate $X$, provided that the member of the interest group $j$ derives greater utility from candidate $X$'s policy platform relative to $Y$'s, that is, $u_j(t_X) > u_j(t_Y)$.*

Mathematically we can write,

$$\frac{dC_j^X}{d\delta} = \frac{h\psi\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi} > 0.$$

The intuition for this result is straightforward. A larger proportion of uninformed voters in the economy, that is, those voters who are not a part of any lobby, entails greater campaign spending which is needed to influence them to vote for a certain candidate.[13] This, in turn, requires more funds to be donated to the candidates by the members of interest groups, because with a rise in the proportion of uninformed people who can be swayed by campaign spending, the effectiveness of money as a tool to attract voters also rises.

### 5.1.5 Comparative Statics with respect to Lobby Organizational Strength Parameter, $O_j$ $(j = L, H)$

**Proposition 8:** *If a member of an interest group $j \in \{L, H\}$ derives greater utility from candidate $X$'s policy platform relative to $Y$'s (that is, $u_j(t_X) > u_j(t_Y)$), then an increase in the organizational ability of that lobby group $j$ ($O_j$), has an ambiguous impact on its individual campaign contributions to electoral candidate $X$.*

This can be explained mathematically as follows. Since,

$$C_j^X = \text{Max}\left\{0, \frac{h\psi\delta\beta_X O_j \eta_j [u_j(t_X) - u_j(t_Y)]}{\phi}\right\}.$$

We will first derive this result for an individual belonging to group $L$. The contribution by an individual member of lobby group $L$ to candidate $X$ can be written as follows:

$$C_L^X = \frac{h\psi\delta\beta_X [u_L(t_X) - u_L(t_Y)]}{\phi} O_L \eta_L . \sum_j \eta_j \phi_j (1 - O_j),$$

where $\sum_j \eta_j \phi_j (1 - O_j) = \delta$. Now differentiating $C_L^X$ with respect to $O_L$, we get,

$$\frac{dC_j^X}{dO_L} = \frac{h\psi\beta_X [u_L(t_X) - u_L(t_Y)]}{\phi} . \eta_L \{\delta - \eta_L \phi_L O_L\} \lesseqgtr 0.$$

[13] Baron (1994) considers the case of particularistic policies in his model, and derives a result similar to this one.

From the above, it can be deduced that the sign of $\frac{dC_j^X}{dO_L}$ depends on the sign of $(\delta - \eta_L \phi_L O_L)$. This means that a rise in the organizational strength of a lobby group $L$ has an ambiguous effect on individual campaign donations provided to electoral candidate $X$, given that $u_L(t_X) > u_L(t_Y)$. This happens due to three different effects at work here:

i) Uninformed (or unorganized) voter effect ($\delta$);

ii) Organizational effect of lobby $L$ ($\eta_L O_L$);

iii) Swing voter effect of voter group $L$ ($\phi_L$).

Therefore, $\frac{dC_j^X}{dO_L} > 0$, if, $\frac{\delta}{\eta_L O_L} > \phi_L$, and, $\frac{dC_j^X}{dO_L} < 0$, if, $\frac{\delta}{\eta_L O_L} < \phi_L$. In words, if the proportion of uninformed voters in the economy who can be swayed by campaign spending ($\delta$) relative to the proportion of organized voters who belong to lobby $L$ ($\eta_L O_L$) is quite small as compared to group $L$'s ideological density, that is, the number of swing voters in group $L$, then the individual members of lobby $L$ will refrain from providing greater campaign donations to candidate $X$ even when there is an exogenous increase in the organizational strength of lobby group $L$ ($O_L$). This is because, $X$ requires contributions to sway uninformed (or unorganized) voters of the economy in her favour, and if the latter are lesser in number in comparison to the number of swing voters in group $L$, these contributions lose their effectiveness as a tool for raising candidate $X$'s vote share. Instead, candidate $X$ would now prefer to target the swing voters in group $L$ with unilateral policy deviations in her equilibrium platform. Therefore, in this scenario, a rise in the organizational ability of a lobby group would result in lower financial resources being transferred to the electoral candidate in the form of donations for campaign expenditure. In contrast, if the proportion of uninformed (or unorganized) voters relative to the proportion of organized voters outweighs the swing voter effect in a group, then an increase in the organizational strength of the lobby in question will lead to an increase in campaign contributions to the electoral candidate. A similar result can be derived for lobby group $H$.

### 5.2 Comparative Statics for Equilibrium Tax Platforms

In this sub-section, we carry out comparative statics for the equilibrium tax platforms of electoral candidates with respect to various parameters, namely, the difference between payoff received with winning and losing an election $(R - Q)$, the effectiveness of campaign spending $(h)$, political candidate $K$'s corruption parameter $(\beta_K)$, the policy preference parameter $(\theta_j)$, the popularity shock variable $(\psi)$, the proportion of people influenced by campaign spending $(\delta)$ and the ideological density of a voter group $j$ $(\phi_j)$. Before moving further, it is important to discuss another result $(R2)$ of our analysis which will be used extensively for analyzing the findings of comparative statics in this sub-section.

*Result R2: In an election scenario comprising of both swing voters and opposing lobby groups,*

*an office-seeking electoral candidate, while choosing her economic policy platform, encounters two key forces; one, the relative organizational strength of lobbies effect* $\left(\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2}\right)$*, and two, the relative swing voter effect* $\left(\frac{\eta_L\phi_L}{\eta_H\phi_H}\right)$.

This result emerges from the derivations worked out for comparative statics of this sub-section. Mathematically, the relative organizational strength of lobbies effect is denoted by $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2}$, while, the relative swing voter effect is denoted by $\frac{\eta_L\phi_L}{\eta_H\phi_H}$ (see Appendix for a detailed derivation). Given that, there exist two voter groups in an economy $L$ and $H$ with different bliss points for economic policy (public good provision) and distinct ideological preferences, some voters from each group may organize and form a lobby to represent their economic interests in a stronger manner via provision of campaign donations to electoral candidates. Since the electoral candidates in our model are completely opportunistic in nature, they now have to choose a policy platform that will help them garner maximum votes to win the election. In order to do this, the candidates have to carefully evaluate the gain and loss in their respective vote shares on account of a deviation in their policy platforms and, hence, strategically choose the policy which gives them the maximum return. Thus, in this setting, the adjustments in equilibrium economic policy (tax) platform by an electoral candidate relies broadly on which of the above effect dominates.

The above result $R2$ provides a fundamental basis for analyzing the findings of following comparative statics.

### 5.2.1 Comparative Statics with respect to Election Payoff Parameter, $(R-Q)$

**Proposition 9:** *Under electoral competition with swing voters and interest groups, the level of public good provision (and tax) in equilibrium rises with an increase in the payoff of winning the election as denoted by* $(R-Q)$*, provided the following sufficiency condition holds:* $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} > \frac{\eta_L\phi_L}{\eta_H\phi_H}$. *The opposite holds true when* $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} < \frac{\eta_L\phi_L}{\eta_H\phi_H}$.

The mathematical proof of this result is relegated to Appendix. By utilizing result $R2$, we can say that, if $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} > \frac{\eta_L\phi_L}{\eta_H\phi_H}$, then $\frac{dt_X^e}{d(R-Q)} > 0$ and if, $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} < \frac{\eta_L\phi_L}{\eta_H\phi_H}$, then $\frac{dt_X^e}{d(R-Q)} < 0$. When the payoff from winning the election $(R-Q)$ rises, it provides the electoral candidate an incentive to shift the tax platform in favour of the voter group that has a larger number of swing voters and/or in favour of the strongly organized interest group. From the first condition we can see that, if the interest group $L$ is relatively better organized than interest group $H$, the tax platform chosen by the electoral candidate rises (that is, moves in favour of the $H$ group). This could be attributed to the fact that a well-organized lobby group entails that its corresponding voter group must comprise of a lower proportion of swing or ideologically neutral voters, that is, those voters who would not have any incentive to make donations in order to become a part of an interest group and lobby for a specific policy or cause. Thus, from our result we can infer that,

in an economy with two economically distinct voter groups whose respective policy bliss points are endorsed by their corresponding lobby groups, the policy platform of a purely opportunistic electoral candidate moves in favour of the voter group consisting of a larger proportion of swing voters when the payoff from winning the election increases. In essence, we find that, higher stakes of electoral success in terms of holding office and attaining "ego" rents, induces the opportunistic candidate in our model to concentrate on targeting the swing voters from each voter group and secure her vote share, rather than bartering a lobby group's monetary support for a favourable deviation in equilibrium policy choice, in turn, losing out major share of voters.

### 5.2.2 Comparative Statics with respect to Campaign Spending Efficiency Parameter, $h$

**Proposition 10:** *An increase in the effectiveness of campaign contributions as a vote seeking instrument (denoted by $h$), tilts an electoral candidate's equilibrium tax platform towards the preferences of group $H$ or $L$ according as:* $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} \lessgtr \frac{\eta_H\phi_H}{\eta_L\phi_L}$.

The proof of this proposition is left to Appendix. Again from result $R2$, there are two main effects at work: the relative organizational strength of lobbies effect and the relative swing voter effect. More specifically, $\frac{dt_X^e}{dh} > 0$ if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} > \frac{\eta_H\phi_H}{\eta_L\phi_L}$ and $\frac{dt_X^e}{dh} < 0$ if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} < \frac{\eta_H\phi_H}{\eta_L\phi_L}$. Intuitively speaking, the electoral candidate will raise her tax platform (in favour of the economic preferences of voter group $H$) if the interest group $H$ has a greater organizational strength relative to interest group $L$. Because of a rise in the vote-garnering efficiency of monetary campaign donations that are provided to the electoral candidate by the interest groups, the importance attached to the role of interest groups by the electoral candidate is enhanced in comparison to the importance attached to the targeting of swing voters. Thus, in this case, the political candidate will adjust her equilibrium policy position in favour of the interest group that has a greater relative organizational strength.

### 5.2.3 Comparative Statics with respect to Honesty Parameter, $\beta_K$ $(K = X, Y)$

**Proposition 11:** *If an electoral candidate $X$ is sufficiently corrupt in respect of spending of campaign money (that is, $\beta_X < 0.5$), then she is more inclined to sway her policy platform in favour of the interest group that is relatively stronger in terms of organizational strength even after an increase in $\beta_X$. However, if candidate $X$ is sufficiently honest (that is, $\beta_X > 0.5$), the impact of a further rise in $\beta_X$ on equilibrium tax platform turns out to be ambiguous. Similar results hold for political candidate $Y$.*

The mathematical proof of this result has been provided in Appendix. As mentioned earlier, in our model, $\beta_X = 0.5$ denotes the extent of honesty of electoral candidate $X$ indicating that half of the contribution money received by her is spent on campaign advertisements to woo the voters

and the other half is kept for her private use. A value of $\beta_X$ less than 0.5 implies that $X$ keeps a greater proportion of the donation money for her private use and spends a lower proportion on voters, thereby signalling a corrupt or dishonest nature of the politician. In contrast, $\beta_X$ greater than 0.5 implies that $X$ keeps a lesser proportion of the donation money for her private use and spends a higher proportion on voters, thus indicating a more honest nature of the politician. We find that, when an electoral candidate transitions from being highly corrupt to being relatively more honest (that is, $\beta_X$ rising but still being less than the threshold value of 0.5), the relative lobby organizational effect is found to dominate the relative swing voter effect (see Result $R2$). This is because, even though the electoral candidate is now comparatively more honest, still she is not able to outbalance the significance that she attaches to the monetary donations procured from lobby groups when she is relatively more dishonest and, therefore, attempts to divert more campaign funds towards personal use. For the purpose of meeting the campaign expenditures to sway voters, the candidate thus needs to shift her equilibrium policy platform in favour of the preferences of the relatively well organized interest group. In sum, provided that the political candidate is corrupt, if the lobby group $H$ is better organized, then public good provision will increase in conformity with the former's economic preferences, while if lobby group $L$ is stronger, then the tax platform will reduce implying an adjustment towards the $L$ group's public good preferences. Alternately, if the degree of honesty of an electoral candidate is quite high to begin with, then a further rise in her honesty will have an ambiguous influence on her equilibrium tax choice (see Appendix for a mathematical explanation).

#### 5.2.4 Comparative Statics with respect to Policy Bliss Point Parameter, $\theta_j$ $(j = L, H)$

**Proposition 12:** *An electoral candidate $X$ increases (decreases) the level of her equilibrium tax platform in response to a rise (fall) in the voter group $j$'s economic preference parameter for public good provision (denoted by $\theta_j$). Similar result holds for electoral candidate $Y$.*

To prove this result, we differentiate equation (26) with respect to $\theta_j$, to get,

$$\frac{dt_X^e}{d\theta_j} = \frac{(R-Q)\left[\eta_j\phi_j + \frac{\psi}{\phi}h^2\beta_X^2\delta^2 O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta O_j^2\eta_j^2]}{n(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]} > 0.$$

Given that $R > Q$ and $0 \leq \beta_X \leq 1$, $\frac{dt_X^e}{d\theta_j} > 0$. This means that, if in an economy, voters on the whole have a greater preference for public good (and hence tax), it subsequently results in a higher tax platform being chosen by the electoral candidate in an equilibrium involving the presence of both swing voters and interest groups. This is because, the electoral candidates in our model are purely office-seeking and have no economic preferences of their own. They, therefore, will move their respective policy platforms (tax and public good provision) towards the average of the economic bliss points of all voter groups, which would be at a relatively higher level in order

to maximize their chances of winning the election. In contrast, if the majority of voters prefer a lower level of public good, then the corresponding tax platform (as well as the level of public good provision) as promised by the opportunistic electoral candidates would be also be lower. To this end, we can say that the centripetal force (that is, a move towards the median policy by the electoral candidate) plays a major role in deciding the equilibrium policy choice in this case.

### 5.2.5 Comparative Statics with respect to Popularity Parameter, $\psi$

**Proposition 13:** *An increase in the popularity density of an electoral candidate (denoted by $\psi$), biases her equilibrium policy (tax) choice towards that of the stronger lobby's bliss point irrespective of which voter group has a greater number of swing voters.*

Mathematical proof of this result is relegated to Appendix. The intuition for this proposition again rests on result $R2$. In our model, an electoral candidate's average popularity can be raised among the voters either through a stochastic positive popularity shock or through an increase in electoral campaign expenditures. With an increase in popularity density across the voter groups, an electoral candidate knows that she can exploit a substantial vote base and increase her chances of electoral win by spending more on electoral campaign and becoming relatively more popular. However, greater campaign spending entails significant monetary donations that need to be secured from the interest groups. Consequently, the political candidate has to move her equilibrium policy point closer to that of the relatively stronger lobby group's bliss point in return for greater financial assistance. It should be noted that, this policy deviation by the electoral candidate towards the stronger lobby's bliss point would generate both a gain and a loss in vote share. The loss would accrue from the voter group supported by the weaker lobby, while the gain in vote share would accrue from the voter group supported by the stronger lobby group. This loss in vote share, however, would not be substantial for the electoral candidate on account of the gain in votes that she garners due to a rise in her average popularity. In sum, we can say that in this case, the effect of relative organizational strength of lobby groups on equilibrium tax platform outweighs the effect of swing voters on equilibrium policy choice.

### 5.2.6 Comparative Statics with respect to Proportion of Uninformed Voters, $\delta$

**Proposition 14:** *An electoral candidate ($X$ or $Y$) tilts her equilibrium policy (tax) platform in favour of the bliss point of a relatively well organized interest group when the proportion of uninformed (or unorganized) voters (depicted by $\delta$) rise in the economy.*

From the proof of this result presented in Appendix, we can infer that $\frac{dt_X^e}{d\delta} > 0$ if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} > \frac{\eta_H\phi_H}{\eta_L\phi_L}$ and $\frac{dt_X^e}{d\delta} < 0$ if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} < \frac{\eta_H\phi_H}{\eta_L\phi_L}$. The intuition for this result is analogous to the one presented in the previous proposition. More specifically, uninformed voters are those who can be influenced by an

electoral candidate with greater campaign advertisement expenditures, that are financed from the monetary contributions garnered from interest groups. The electoral candidate, therefore, knows that she can increase her vote share and probability of winning by strategically deviating her equilibrium policy platform in favour of the stronger lobby group's bliss point in order to get more donation money in return and spending it on the uninformed voters, whose relative proportion in the economy's total population has now increased. Hence, similar to the previous finding, the lobby's relative organizational strength effect dominates the swing voter effect (as explained in result $R2$) in this scenario as well.

### 5.2.7 Comparative Statics with respect to Ideological Density Parameter, $\phi_j$ ($j = L, H$)

**Proposition 15:** *An increase in the ideological density of a voter group (say, $L$) leads to a shift in an electoral candidate's ($X$ or $Y$) equilibrium policy (tax) platform towards that of group $L$'s preferred policy point, if the following sufficiency conditions hold:* $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} < \frac{\eta_L\phi_L}{\eta_H\phi_H} < \left(\frac{\frac{\theta_H}{n}-\theta_L}{\theta_L-\frac{\theta_L}{n}}\right)$ *and* $\frac{\theta_H}{n} > \theta_L$*, whereas, it will shift towards group $H$'s preferred policy point, if the following sufficiency conditions hold:* $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} > \frac{\eta_L\phi_L}{\eta_H\phi_H} > \left(\frac{\frac{\theta_H}{n}-\theta_L}{\theta_L-\frac{\theta_L}{n}}\right)$ *and* $\frac{\theta_H}{n} < \theta_L$.

The mathematical proof of this proposition is provided in Appendix. In the given sufficiency condition, we can define $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2}$ as the relative organizational strength of group $L$ and $\frac{\eta_L\phi_L}{\eta_H\phi_H}$ as the proportion of swing voters in group $L$ relative to group $H$ (refer to result $R2$). Additionally, $\left(\frac{\frac{\theta_H}{n}-\theta_L}{\theta_L-\frac{\theta_L}{n}}\right)$ can be defined as the deviation of the first-best tax solution $\left(\frac{\theta_H}{n}\right)$ for group $H$ from group $L$'s bliss point in proportion to the deviation of the bliss point of the $L$ group from its first-best tax solution $\left(\frac{\theta_L}{n}\right)$. As $\phi_L$ rises, the proportion of swing voters in $L$ voter group rises (see result $R1$) and this swing voter effect persuades the electoral candidate to choose a tax platform closer to group $L$'s policy bliss point. Our sufficiency condition shows that the relative swing voter effect in group $L$ has to outweigh the relative organizational strength of lobbies effect of group $L$ for the tax platform to fall with an increase in ideological density of group $L$. However, this sufficiency condition will only hold if $\frac{\theta_H}{n} > \theta_L$ holds, which indicates that the voters in group $L$ should have a much lower preference for public good (and hence, a lower tax level) as compared to the first-best tax policy preferred by group $H$ voters. These conditions imply that, given such well-defined policy preferences of both voter groups, that is, when the voter groups are quite distinct from each other in terms of their respective economic preferences along with voter group $H$ having the support of a relatively stronger lobby group ($H$), an electoral candidate's tax platform would still deviate towards group $L$'s bliss point as a result of an increase in the ideological density of group $L$. The reason is that, under these stringent conditions the swing voter effect (in voter group $L$) would be very strong and, thus, an increase in group $L$'s ideological density will compensate

the electoral candidate adequately for her loss in vote share from voter group $H$ on account of a reduction in her equilibrium tax platform level, thereby, raising her likelihood of winning the election.

In contrast, when $\theta_L > \frac{\theta_H}{n}$, voters in group $L$ can be understood to have a greater preference for public good provision in relation to voter group $H$'s first-best equilibrium tax choice. This condition reveals that the voters in groups $L$ and $H$ have quite similar economic policy (public good) preferences and, therefore, cannot be considered as greatly divergent on the economic policy scale. In this situation, the electoral candidate finds it easier to raise her equilibrium tax platform, given that lobby group $L$ has a much stronger organizational capacity relative to lobby $H$, since both lobby groups $L$ and $H$ contest for a relatively higher level of public good provision. Notwithstanding the fact that with a relatively greater ideological density, voter group $L$ has a greater proportion of swing voters now, which when assessed in terms of loss in vote share would be quite substantial, the electoral candidate considers it appropriate to increase her equilibrium tax platform. It is evident that this rise in equilibrium tax platform and, hence, the level of public good provision would lead to a gain in candidate's vote share from group $H$ and a loss in vote share from group $L$. Nevertheless, in accordance with our sufficiency condition, provided that the relative organizational strength of interest group $L$ overpowers the relative swing voter effect in voter group $L$, and that the voters from both groups have analogous economic preferences, an unequivocal compensation of the loss in the candidate's vote share from group $L$ via a gain in her vote share from across voter groups cannot be ruled out.

# 6 Conclusion

This paper constructed a stylized model of election between two opportunistic candidates who can influence equilibrium policy platforms in exchange for monetary contributions provided by two distinct lobby groups. Two key features are embedded in this model, one, the presence of ideologically neutral or swing voters is accounted for in the modelling framework, and two, the electoral candidates embezzle a proportion of funds they receive from interest groups. Both these attributes create a dual uncertainty within the model. The first is related to the random factors that can potentially affect voter's decisions to vote for a certain candidate, which entails that electoral candidates in our model have incomplete information about voter's preferences. The second uncertainty arises on part of lobby groups who are unsure whether an electoral candidate will honestly utilize their contributions to increase their chances of electoral success. We compare the equilibrium policy choices of the two opportunistic candidates in the scenario where none of the above uncertainties exist (the benchmark case), where only uncertainty about voter's preferences exist (swing voter case), and where both these uncertainties exist (case where both swing voters

and lobby groups exist).

From our results, we found that an opportunistic candidate's tax platform in the swing voter case is always lower than the tax platform of the same candidate in the benchmark case. This indicates that the presence of swing voters in the economy effectively dilutes the preference of public good provision for each group as a whole, and consequently, the level of public good provision is also reduced as compared to the public good provision under the benchmark scenario. Furthermore, the level of public good provision (and the associated tax policy) is found to be higher in a scenario involving both swing voters and interest groups as compared to the benchmark case if, both the organizational strength as well as the ideological density (as compared to average ideology of the population) of a voter group favouring a higher level of public good provision is greater than the organizational strength and ideological density (as compared to average ideology of the population) of the voter group favouring a lower level of public good provision.

In addition to this, a comparison of the equilibrium tax platform in the presence of both interest groups and voter groups with differing ideological densities with the equilibrium tax platform under swing voter case illustrates that the tax platform under the former scenario will be higher (lower) than the tax platform under the latter scenario if the preference for public good provision of both voter groups is greater (lower) than the actual public good provision under the swing voter case. Moreover, if one voter group in the economy prefers more public good relative to the actual level of provision while the other group prefers relatively less of it, then the introduction of two such antithetical interest groups in the swing voter case would create contradictory forces at work, where, one interest group will lobby for an increase in the public good provision while the other will lobby for a reduction in the level of public good provision. Hence, the equilibrium tax choices of electoral candidates in the presence swing voters and opposing lobby groups will adjust accordingly to whichever of the two lobby groups is stronger in terms of organizational strength.

Furthermore, the results from comparative statics with respect to equilibrium contributions show that, as the electoral candidate becomes more honest with respect to the spending of the contribution money for electoral campaigning (that is, indulges in lesser leakage of monetary funds), it induces the interest group to raise the level of donations to the candidate. This is because, donation money is now used by an electoral candidate for the purpose of campaign spending, which in turn, is used to influence voters to win elections. If the political candidate does not indulge in greater embezzlement of campaign funds, the marginal benefit derived by the donor interest group would be greater due to smaller leakage of money, which in turn will increase the campaign donations to the electoral candidate. We also find that an increase in the organizational ability of a voter group leads to two opposite impacts on campaign donations by the same group. On one hand, the electoral candidates try to sway the uninformed voters in the economy through campaign spending, and, if the proportion of these uninformed voters is more

relative to the proportion of individuals who are a part of a lobby, an increase in the organizational strength of the lobby results in greater contributions being offered to the electoral candidate. This is the uninformed voter effect. On the other hand, there is the direct organizational strength effect of lobby, which suggests that if a lobby is already strong in terms of its organizational capability relative to the proportion of uninformed voters in the economy, then the lobby members need not put too much effort into providing electoral candidates with greater amounts of donation money. Therefore, the impact of a rise in a lobby group's organizational strength on the campaign donations is ambiguous and depends on which of the above two effects dominate.

We also derive the comparative statics of equilibrium tax platforms with respect to a range of important parameters such as the difference between the payoff received with winning and losing an election, the effectiveness of campaign spending, a political candidate's corruption (or leakage) parameter, policy preference parameter, the popularity shock parameter, the proportion of people influenced by campaign spending and the ideological density of a voter group. For each of these, it has been found that there are two main strategic forces of interaction in our model that shape the equilibrium policy platforms of candidates into what they are, namely the relative swing voter effect and the relative organizational strength of lobbies effect. In sum, we conclude that the equilibrium policy (tax) platform of the electoral candidates sways in favour of the more dominant out of the above two effects and towards the economic policy preferences of the voter group corresponding to the relatively stronger effect.

Although being a highly popular and legalized channel in political democracies, campaign contributions can exhibit substantial distortion especially when they are provided in exchange for policy favours and when they are misappropriated by policy-makers and/or electoral representatives. Our model accounts for both types of distortions and their impact on office-seeking electoral candidate's equilibrium policy choice. However, our model can be extended further by introducing partisan and mixed motivations of electoral candidates, wherein, they also derive utility from different levels of tax and public good provision. This will introduce another centrifugal effect into the present framework as now the candidates would not want to deviate too much from their respective policy bliss points, in turn, limiting the significance of the monetary donations that they garner from the interest groups. Another extension could be the introduction of a binding limit or a cap on the campaign contributions which will have direct implications on the total contributions made by lobbies as well as competition among different lobby groups. It is evident that campaign spending has a two-fold impact on the welfare of uninformed voters. On one hand, greater campaign expenditure helps these voters acquire more information about the politicians they will vote for in an election. On the other hand, it deviates the equilibrium policy choice in favour of the interest groups' policy preferences, which is not suitable for the general electorate. Therefore, an assessment of the welfare consequences of campaign spending and embezzlement in

the post-election period also lays the foundation for future work on this interesting topic.

## References


Austen-Smith, D. (1987) Interest Groups, Campaign Contributions and Probabilistic Voting. *Public Choice* 54: 123-139.

Baron, D.P. (1988) Regulation and Legislative Choice. *The RAND Journal of Economics* 19: 467-477.

Baron, D.P. (1994) Electoral Competition with Informed and Uniformed Voters. *American Political Science Review* 88: 33-47.

Besley, T., Coate S. (1997) An Economic Model of Representative Democracy. *Quarterly Journal of Economics* 112: 85-114.

Besley, T., Coate S. (2001) Lobbying and Welfare in a Representative Democracy. *Review of Economic Studies* 68: 67-82.

Bruter, M., Harrison, S. (2007) United Kingdom, In Deloye, Y. and Bruter, M. (eds.) *Encyclopaedia of European Elections*. Palgrave Macmillan, Basingstoke.

Coughlin, P.J., Mueller D.C., Murrell P. (1990) A Model of Electoral Competition with Interest Groups. *Economics Letters* 32: 307-311.

Cox, G.W. (1990) Centripetal and Centrifugal Incentives in Electoral Systems. *American Journal of Political Science* 34: 903-935.

Damania, R., Yalcin E. (2008) Corruption and Political Competition. Economics Discussion Paper No. 2008-7. http://dx.doi.org/10.2139/ssrn.1726705.

Downs, A. (1957) An Economic Theory of Democracy. Harper and Row, New York.

Frendreis, J., Gitelson, A.R., Jenkins S., Roscoe, D.D. (2003) Testing Spatial Model of Elections: The Influence of Voters and Elites on Candidate Issue Positions. *Legislative Studies Quarterly* 28: 77-101.

Grofman, B. (2004) Downs and Two-Party Convergence. *Annual Review of Political Science* 7: 25-46.

Groseclose, T., Snyder, J.M. Jr. (1996) Buying Supermajorities. *American Political Science Review* 90: 303-315.

Grossman, G.M., Helpman E. (1996) Electoral Competition and Special Interest Politics. *Review of Economic Studies* 63: 265-286.

Grossman G.M., Helpman E. (2005) A Protectionist Bias in Majoritarian Politics. *The Quarterly Journal of Economics* 120: 1239-1282.

Hall, R.L., Deardorff A.V. (2006) Lobbying as Legislative Subsidy. *American Political Science Review* 1: 69-84.

Le, T., Yalcin E. (2018) Lobbying, Campaign Contributions and Electoral Competition. *European Journal of Political Economy* 55: 559-572.

Mayer, W., Li, J. (1994) Interest Groups, Electoral Competition, And Probabilistic Voting For Trade Policies. *Economics and Politics* 6: 59-77.

McCarty, N., Poole, K.T., Rosenthal, H. (2006) Polarized America: The Dance of Ideology and Unequal Riches. MIT Press, Cambridge.

Morton, R.B., Myerson R.B. (1992) Campaign Spending with Impressionable Voters. Discussion Paper 1023, Northwestern University, Center for Mathematical Studies in Economics and Management Science. https://www.kellogg.northwestern.edu/research/math/papers/1023.pdf

Persson, T., Helpman E. (1998) Lobbying and Legislative Bargaining. NBER Working Paper No. 6589. https://www.nber.org/papers/w6589

Persson, T., Tabellini G. (2002) Political Economics: Explaining Economic Policy. MIT Press, Cambridge.

Prat, A. (2002) Campaign Spending with Office Seeking Politicians, Rational Voters, and Multiple Lobbies. *Journal of Economic Theory* 103: 162-189.

Redoano, M. (2010) Does Centralization Affect the Number and Size of Lobbies? *Journal of Public Economic Theory* 12: 407-435.

Stonecash, J.M., Brewer, M.D., Mariani, M.D. (2003) Diverging Parties: Social Change, Realignment and Party Polarization. Westview Press, Boulder, Colorado.

Wilson, J.K., Damania, R. (2005) Corruption, Political Competition and Environmental Policy. *Journal of Environmental Economics and Management* 49: 516-535.

# Appendix

***Proof of Proposition 9:*** We focus first on the mathematical derivation for electoral candidate $X$. Using a similar logic, the same results can also be obtained for candidate $Y$. Differentiating equation (26) with respect to $(R-Q)$, we get,

$$\frac{dt_X^e}{d(R-Q)} = \frac{[(1-\beta_X)\beta_X h\delta]\left[t_X^s \sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lesseqgtr 0.$$

As can be seen from the above, the sign of the derivative depends upon the sign of the following term: $\left(t_X^s \sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right)$. Now by using equation (16) to expand this term, we get,

$$\left(t_X^s \sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right) = \frac{(\theta_H-\theta_L)}{n}(O_L^2\eta_L^2\eta_H\phi_H) - (O_H^2\eta_H^2\eta_L\phi_L)\frac{(\theta_H-\theta_L)}{n},$$

or,

$$\left(t_X^s \sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right) = \frac{(\theta_H-\theta_L)}{n}[O_L^2\eta_L^2\eta_H\phi_H - O_H^2\eta_H^2\eta_L\phi_L] \lesseqgtr 0.$$

Given our assumption that $\theta_H > \theta_L$, the above term is positive if $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} > \frac{\eta_L\phi_L}{\eta_H\phi_H}$. And this term is negative if, $\frac{O_L^2\eta_L^2}{O_H^2\eta_H^2} < \frac{\eta_L\phi_L}{\eta_H\phi_H}$.

***Proof of Proposition 10:*** Here, we show the result for electoral candidate $X$. The result can be replicated for candidate $Y$. Differentiating equation (26) with respect to $h$, we get that,

$$\frac{dt_X^e}{dh} = \frac{(R-Q)\beta_X\delta\left[2(R-Q)\frac{\psi}{\phi}h\beta_X\delta + (1-\beta_X)\right]\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lesseqgtr 0.$$

Given that $R > Q$ and that $0 \leq \beta_X \leq 1$, the sign of $\frac{dt_X^e}{dh}$ will depend upon the sign of the term $\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]$. Using equation (16) to expand the above term, we get,

$$\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right] = \frac{(\theta_H-\theta_L)}{n}[O_H^2\eta_H^2\eta_L\phi_L - O_L^2\eta_L^2\eta_H\phi_H] \lesseqgtr 0.$$

***Proof of Proposition 11:*** This can be proved mathematically by differentiating equation (26) with respect to $\beta_X$:

$$\frac{dt_X^e}{d\beta_X} = \frac{[(R-Q)h\delta]\left[2(R-Q)\frac{\psi}{\phi}h\beta_X\delta + (1-2\beta_X)\right]\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lesseqgtr 0.$$

Using the equilibrium swing voter tax solution from equation (16) to expand the above expression, we get,

$$\frac{dt_X^e}{d\beta_X} = \frac{[(R-Q)h\delta]\left[2(R-Q)\frac{\psi}{\phi}h\beta_X\delta + (1-2\beta_X)\right](\theta_H-\theta_L)[O_H^2\eta_H^2\eta_L\phi_L - O_L^2\eta_L^2\eta_H\phi_H]}{n\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2}.$$

Given $R > Q$, it can be deduced from the above that,

$$\frac{dt_X^e}{d\beta_X} > 0 \text{ if } \frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} > \frac{\eta_H\phi_H}{\eta_L\phi_L} \text{ and } \beta_X \leq 0.5.$$

And,

$$\frac{dt_X^e}{d\beta_X} < 0 \text{ if } \frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} < \frac{\eta_H\phi_H}{\eta_L\phi_L} \text{ and } \beta_X \leq 0.5.$$

In contrast, when $\beta_X > 0.5$, then we cannot be sure about the impact of a rise in $\beta_X$ on the equilibrium tax platform as can be seen from the following expression:

$$\frac{dt_X^e}{d\beta_X} = \frac{\overbrace{[(R-Q)h\delta]\left[\overbrace{2(R-Q)\frac{\psi}{\phi}h\beta_X\delta}^{+} + \overbrace{(1-2\beta_X)}^{-}\right]}^{+/-}\overbrace{(\theta_H-\theta_L)}^{+}\overbrace{[O_H^2\eta_H^2\eta_L\phi_L - O_L^2\eta_L^2\eta_H\phi_H]}^{+/-}}{\underbrace{n\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2}_{+}}.$$

The uncertain impact of a change in $\beta_X$ on the equilibrium policy choice is apparent from the ambiguous signs of the expressions in the numerator of the above equation. For electoral candidate $Y$, a similar result can be derived by differentiating equation (27) with respect to $\beta_Y$.

***Proof of Proposition 13:*** This can be explained mathematically as follows. We differentiate equation (26) with respect to $\psi$ to get,

$$\frac{dt_X^e}{d\psi} = \frac{(R-Q)^2\frac{h^2}{\phi}\beta_X^2\delta^2\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lessgtr 0.$$

Now, $\frac{dt_X^e}{d\psi} > 0$ if $\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right] > 0$ and $\frac{dt_X^e}{d\psi} < 0$ if $\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right] < 0$. That is, $\frac{dt_X^e}{d\psi} > 0$ if:

$$\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right] = \frac{(\theta_H-\theta_L)}{n}[O_H^2\eta_H^2\eta_L\phi_L - O_L^2\eta_L^2\eta_H\phi_H] > 0,$$

or, $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} > \frac{\eta_H\phi_H}{\eta_L\phi_L}$ because $\theta_H > \theta_L$. In contrast, if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} < \frac{\eta_H\phi_H}{\eta_L\phi_L}$ then $\frac{dt_X^e}{d\psi} < 0$.

***Proof of Proposition 14:*** This can be explained mathematically as follows by differentiating equation (26) with respect to $\delta$, to get,

$$\frac{dt_X^e}{d\delta} = \frac{(R-Q)\beta_X h\left[2(R-Q)\frac{\psi}{\phi}h\beta_X\delta + (1-\beta_X)\right]\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lesseqgtr 0.$$

Again expanding the term $\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right]$ by using equation (16), we get,

$$\left[\phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n} - t_X^s\sum_j O_j^2\eta_j^2\right] = \frac{(\theta_H-\theta_L)}{n}[O_H^2\eta_H^2\eta_L\phi_L - O_L^2\eta_L^2\eta_H\phi_H] \lesseqgtr 0.$$

Given that $R > Q$ and $\theta_H > \theta_L$, we can say that $\frac{dt_X^e}{d\delta} > 0$ if $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} > \frac{\eta_H\phi_H}{\eta_L\phi_L}$ and $\frac{dt_X^e}{d\delta} < 0$ if, $\frac{O_H^2\eta_H^2}{O_L^2\eta_L^2} < \frac{\eta_H\phi_H}{\eta_L\phi_L}$.

***Proof of Proposition 15:*** This can be explained mathematically for electoral candidate $X$ by differentiating equation (26) with respect to $\phi_j$ which yields the following expression:

$$\frac{dt_X^e}{d\phi_j} = \frac{(R-Q)^2\eta_j[\phi\theta_j - t_X^s] + \left[\theta_j\sum_j O_j^2\eta_j^2 - \frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]\left[(R-Q)^2\eta_j\frac{\psi}{\phi}h^2\beta_X^2\delta^2 + (R-Q)[(1-\beta_X)\beta_X h\delta\eta_j\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2}$$
$$+\frac{(R-Q)^2\left[\frac{\psi}{\phi^2}\eta_j h^2\beta_X^2\delta^2\right]\left[t_X^s\sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]}{\{(R-Q)\left[\phi + \frac{\psi}{\phi}h^2\delta^2\beta_X^2\sum_j O_j^2\eta_j^2\right] + [(1-\beta_X)\beta_X h\delta\sum_j O_j^2\eta_j^2]\}^2} \lesseqgtr 0.$$

It is apparent from the above expression that the sign of $\frac{dt_X^e}{d\phi_j}$ depends upon the terms $[\phi\theta_j - t_X^s]$, $\left[\theta_j\sum_j O_j^2\eta_j^2 - \frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]$ and $\left[t_X^s\sum_j O_j^2\eta_j^2 - \phi\frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right]$. To further analyze these terms, we focus on the ideological density variable for the $L$ voter group, that is, $\phi_L$. Now, using the definition of average ideology discussed earlier in the paper and equation (16), expansion of the first term results in the following:

$$[\phi\theta_L - t_X^s] = \left[\theta_L(\eta_L\phi_L + \eta_H\phi_H) - \frac{\eta_L\phi_L\theta_L}{n} - \frac{\eta_H\phi_H\theta_H}{n}\right],$$

$$[\phi\theta_L - t_X^s] = \left[\eta_L\phi_L\left(\theta_L - \frac{\theta_L}{n}\right) + \eta_H\phi_H\left(\theta_L - \frac{\theta_H}{n}\right)\right]. \tag{34}$$

As for the second term, we can write,

$$\left[\theta_j\sum_j O_j^2\eta_j^2 - \frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] = \left[\theta_L(O_L^2\eta_L^2 + O_H^2\eta_H^2) - \frac{O_L^2\eta_L^2\theta_L}{n} - \frac{O_H^2\eta_H^2\theta_H}{n}\right],$$

or,

$$\left[\theta_j\sum_j O_j^2\eta_j^2 - \frac{\sum_j O_j^2\eta_j^2\theta_j}{n}\right] = \left[O_L^2\eta_L^2\left(\theta_L - \frac{\theta_L}{n}\right) + O_H^2\eta_H^2\left(\theta_L - \frac{\theta_H}{n}\right)\right]. \tag{35}$$

And finally, by using equation (16) the third term can be expanded and its further simplification yields:

$$\left[t_X^s \sum_j O_j^2 \eta_j^2 - \phi \frac{\sum_j O_j^2 \eta_j^2 \theta_j}{n}\right] = \frac{(\theta_H - \theta_L)}{n} \eta_L \eta_H [O_L^2 \eta_L \phi_H - O_H^2 \eta_H \phi_L]. \tag{36}$$

From expressions (34), (35) and (36), we infer that $\frac{dt_X^e}{d\phi_L} < 0$, if the following sufficiency conditions hold: $\frac{\eta_L \phi_L}{\eta_H \phi_H} < \left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right)$ and $\frac{O_L^2 \eta_L^2}{O_H^2 \eta_H^2} < \left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right)$ and $\frac{O_L^2 \eta_L^2}{O_H^2 \eta_H^2} < \frac{\eta_L \phi_L}{\eta_H \phi_H}$. These three conditions can be combined to get the following result:

$$\frac{dt_X^e}{d\phi_L} < 0 \text{ if } \frac{O_L^2 \eta_L^2}{O_H^2 \eta_H^2} < \frac{\eta_L \phi_L}{\eta_H \phi_H} < \left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right).$$

In the above condition, by definition, $\frac{O_L^2 \eta_L^2}{O_H^2 \eta_H^2}$ and $\frac{\eta_L \phi_L}{\eta_H \phi_H}$ are positive terms and since both of them have to be less than the term $\left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right)$ for $\frac{dt_X^e}{d\phi_L} < 0$, this ensures that the term $\left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right)$ should be positive. Now, given that $\theta_H > \theta_L > 0$, this term will be positive only when $\frac{\theta_H}{n} > \theta_L$.

In contrast, using equations (34), (35) and (36), we can say that, $\frac{dt_X^e}{d\phi_L} > 0$ if,

$$[\phi \theta_L - t_X^s] = \left[\eta_L \phi_L \left(\theta_L - \frac{\theta_L}{n}\right) + \eta_H \phi_H \left(\theta_L - \frac{\theta_H}{n}\right)\right] > 0,$$

$$\left[\theta_j \sum_j O_j^2 \eta_j^2 - \frac{\sum_j O_j^2 \eta_j^2 \theta_j}{n}\right] = \left[O_L^2 \eta_L^2 \left(\theta_L - \frac{\theta_L}{n}\right) + O_H^2 \eta_H^2 \left(\theta_L - \frac{\theta_H}{n}\right)\right] > 0,$$

and,

$$\left[t_X^s \sum_j O_j^2 \eta_j^2 - \phi \frac{\sum_j O_j^2 \eta_j^2 \theta_j}{n}\right] = \frac{(\theta_H - \theta_L)}{n} \eta_L \eta_H \left[O_L^2 \eta_L \phi_H - O_H^2 \eta_H \phi_L\right] > 0.$$

All these individual conditions can condensed into the following condition:

$$\frac{O_L^2 \eta_L^2}{O_H^2 \eta_H^2} > \frac{\eta_L \phi_L}{\eta_H \phi_H} > \left(\frac{\frac{\theta_H}{n} - \theta_L}{\theta_L - \frac{\theta_L}{n}}\right).$$

Furthermore, using equation (34), we can see that, since, $\theta_H > \theta_L > 0$, the term $\eta_L \phi_L \left(\theta_L - \frac{\theta_L}{n}\right)$ is positive, and therefore, for $[\phi \theta_L - t_X^s] > 0$, the term $\eta_H \phi_H \left(\theta_L - \frac{\theta_H}{n}\right)$ should also be positive. This will occur when $\theta_L > \frac{\theta_H}{n}$ implying that the voters in group $L$ prefer a higher level of public good provision relative to voter group $H$'s first-best equilibrium tax choice.